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Three-Dimensional Lattice Boltzmann Flux Solver and Its Applications to Incompressible Isothermal and Thermal Flows

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Abstract. A three-dimensional (3D) lattice Boltzmann flux solver (LBFS) is presented in this paper for the simulation of both isothermal and thermal flows. The present solver combines the advantages of conventional Navier-Stokes (N-S) solvers and lattice Boltzmann equation (LBE) solvers. It applies the finite volume method (FVM) to solve the N-S equations. Different from the conventional N-S solvers, its viscous and inviscid fluxes at the cell interface are evaluated simultaneously by local reconstruction of LBE solution. As compared to the conventional LBE solvers, which apply the lattice Boltzmann method (LBM) globally in the whole computational domain, it only applies LBM locally at each cell interface, and flow variables at cell centers are given from the solution of N-S equations. Since LBM is only applied locally in the 3D LBFS, the drawbacks of the conventional LBM, such as limitation to uniform mesh, tie-up of mesh spacing and time step, tedious implementation of boundary conditions, are completely removed. The accuracy, efficiency and stability of the proposed solver are examined in detail by simulating plane Poiseuille flow, lid-driven cavity flow and natural convection. Numerical results show that the LBFS has a second order of accuracy in space. The efficiency of the LBFS is lower than LBM on the same grids. However, the LBFS needs very less non-uniform grids to get grid-independence results and its efficiency can be greatly improved and even much higher than LBM. In addition, the LBFS is more stable and robust.

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Key words: Lattice Boltzmann flux solver, lattice Boltzmann method, Navier-Stokes equation, lid-driven cavity flows, natural convection.

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1 Introduction

In general, for simulation of incompressible fluid flows, the Navier-Stokes (N-S) solver [1–8] and the lattice Boltzmann equation (LBE) solver [9–13] are the two major solvers. Interestingly, these two kinds of solvers are established on different theoretical framework. The N-S solver is based on the direct discretization of the governing equations from macroscopic conservation laws while the LBE solver is based on the solution of the discrete LBE from statistical gas kinetic theory. The roots of both N-S and LBE solvers in their respective theoretical foundations credit themselves unique and distinctive features.

Among various N-S solvers, the commonly applied methods can be roughly classified into three groups: (1) the vorticity-stream-function approach (VSFA) [1–3], (2) the artificial compressibility approach (ACA) [4–6] and (3) the projection approach (PA) [14–16]. The VSFA takes the vorticity and stream function as primary unknowns. The governing equations of the VSFA are reconstructed from the incompressible N-S equations by taking the curl of the momentum equations and introducing the stream function into the calculation of the vorticity. Due to its simplicity and ease in achieving high order of accuracy, the VSFA has attained considerable popularity for simulating two-dimensional (2D) incompressible flows. Unfortunately, due to the intrinsic 2D nature of the stream function, the VSFA is not efficient for three-dimensional (3D) computations. Unlike the VSFA, the ACA directly solves for velocity and pressure. This approach introduces an artificial compressibility term into the continuity equation so that the resultant equation system can be solved in a consistent way. Although it is initially proposed for simulating steady flows, extensions of the ACA for simulation of unsteady flows [6, 17] have also been conducted successfully. Most recently, Asinari et al. [50] proposed a link-wised artificial compressibility method (LW-ACA) by using the simple streaming-collision technologies originated from LBM [11, 12]. The most popular solver in this category is perhaps the PA, which directly solves the incompressible N-S equations. The PA usually introduces a two-stage fractional step technique to resolve the coupling problem between pressure and velocity. The pressure-correction or pressure-Poisson equation is derived and numerically solved to guarantee the divergence-free condition. With the aid of the fractional step technique, the PA attains a rigorous and complete theoretical foundation and does present a simple and accurate solver on Cartesian grids. However, due to the slow convergence of the pressure-Poisson equation, the overall computational efficiency of the PA may be degraded. In addition, as N-S equations are partial differential equations (PDEs), numerical discretization of the first and second order spatial derivatives should be carefully conducted by applying different schemes, which may be tedious and sophisticated for applications on non-uniform grids.

As compared with the N-S solvers, the LBE solvers have emerged as an alternative and powerful algorithm for simulating incompressible flows [13, 18–23]. These solvers are based on mesoscopic kinetic equations and microscopic particle models and take the density distribution functions (DDFs) of each particle as primary unknowns. The macroscopic flow properties are evaluated from the collective behavior of microscopic