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A 3D Multi-Phase Hydrodynamic Model for Cytokinesis of Eukaryotic Cells

Jia Zhao[2] and Qi Wang[1,2,3,*]

[1] *Beijing Computational Science Research Center, Beijing, 100193, P.R. China.*
[2] *Department of Mathematics, University of South Carolina, Columbia, SC 29208, USA.*
[3] *School of Mathematics, Nankai University, Tianjin, 300071, P.R. China.*

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Abstract. In the late stage of the mitotic cycle of eukaryotic cells, cytokinesis ensues during which a parent cell replicates its nucleus with the necessary genetical substances (i.e., DNAs and chromosomes) and splits into two similar offspring cells. This mitotic process involves complex chemical, biophysical and mechanical processes whose details are just beginning to be unfolded experimentally. In this paper, we propose a full 3-D hydrodynamical model using a phase field approach to study the cellular morphological change during cytokinesis. In this model, the force along the contracting ring induced by remodeling of actin-myosin filament on cell cortex layer at the division plane of the parent cell during cytokinesis, is approximated using a proxy force anchored on the newly formed nuclei. The symmetric or asymmetric cell division is simulated numerically with the model. Our numerical results show that the location of the division plane and the contracting force along the cytokinetic ring on the division plane are essential for the cell division. In addition, our numerical study also shows that, during cytokinesis, surface tension of the cell membrane also contributes to this process by retaining the morphological integrity of the offspring cells. This model and the accompanying numerical simulation tool provide a solid framework to build upon with more sophisticated whole cell models to probe the cell mitotic process.

AMS subject classifications: 92C37, 97M10, 76T30

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1 Introduction

A cell is the fundamental unit in all living organisms since animals and plants are all made up of cells of different varieties. The study of cells is therefore an essential part of

*Corresponding author. *Email addresses:* zhao62@math.sc.edu (J. Zhao), qwang@math.sc.edu (Q. Wang)

research in biological science and medicine. Among many functions of a cell, one is the cell's ability of self-reproduction, which is also known as cell proliferation. Owing to cell proliferation, all living organisms can grow and sustain their lives. Given its unique role played in the biological systems, cell study has been the focal point of biology research for centuries. With the advancement of experimental technologies today, more cell functions and micro-structures that facilitate the functions have been uncovered, which reveal an amazingly complex microscopic universe on its own.

One fundamental part of cell study is the cell reproductive cycle, where a parent cell undergoes a sequence of intercellular transformations and eventually divides into two or more offspring cells. For prokaryotic cells, the cell proliferation process is called binary division or binary fission. For eukaryotic cells, it is called mitosis. In the late stage of the cell mitotic process for eukaryotic cells, after the nucleus has been divided and chromosomes separated, the cell division process is also called cytokinesis. For eukaryotic cells, cell division is a much more complicated process than that of prokaryotic cells. It's commonly agreed upon that cell division is not simply driven by a single mechanism, but rather many factors which are intertwined. As one of the most spectacular part of the cell cycle, any single step goes wrong may lead to a failure or even a catastrophe, for instance cancer. Thus, a detailed understanding on cytokinesis can be of great benefits for understanding the source of many diseases. For more details, readers are referred to the insightful review article on this topic in [29]. Readers can also find comprehensive review materials for animal cytokinesis in [9, 13] and for cytokinesis of bacteria in [10]. Besides, some works related to mechanical properties of cells during cytokinesis such as material properties of cells and sources of stresses can be found in [22, 27]. The study published in [11] discusses the molecular requirements for cytokinesis and the work in [2] addresses some recent advances in the mechanism of cytokinesis in animal, yeast and plant cells.

Experimental observations have provided us with a basic picture of cell mitosis. For eukaryotic cells, at the beginning of the cell mitotic process, the parent cell first duplicates its genetic substances and then forms a mitotic spindle consisting of microtubules [21]. Through a cascade of signaling processes [23], the actin and myosin molecules undergo a self-assembly process to remodel the cell cortical layer, a layer rich in actin-filaments located immediately adjacent to the cell membrane [19]. In sync with the elongation of the mitotic spindle, more actin and myosin molecules ascend to a ring like region in a plane roughly orthogonal to the axis of the mitotic spindle to form the cytokinetic ring or contractile ring. The plane is called the cleavage plane or division plane [5]. As more actomyosin molecules are accumulated along the cytokinetic ring, a contracting force is generated and directed inward toward the axis of the spindle [19]. The contracting force pushes the membrane inward to create what is known as the cleavage furrow [27]. The localized activation of the small GTPase Rho family of proteins at the cell division plane controls the position of the contractile ring [30]. When Rho is specifically activated at the division plane within the cortex, it promotes actin polymerization and myosin-2 activation via Rho effector proteins. Rho-GTP promotes actin filament and myosin-2 assembly [6]. The contractile ring is a dynamic structure, in which F-actin and myosin-2