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A Semi-Lagrangian Approach for Dilute Non-Collisional Fluid-Particle Flows

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Abstract. We develop numerical methods for the simulation of laden-flows where particles interact with the carrier fluid through drag forces. Semi-Lagrangian techniques are presented to handle the Vlasov-type equation which governs the evolution of the particles. We discuss several options to treat the coupling with the hydrodynamic system describing the fluid phase, paying attention to strategies based on staggered discretizations of the fluid velocity.

AMS subject classifications: 82C40, 82C80, 65M08

Key words: Semi-Lagrangian methods, particulate flows.

1 Introduction

This paper is concerned with the numerical simulation of dilute suspensions. This study is motivated by many applications ranging from industrial processes to natural flows. For instance, such flows are involved in internal combustion engines and the improvement of their performance need both modeling and computational efforts [50, 56, 71]. The problem is also relevant to fluidized beds [6] where particles are suspended in the fluid stream, in order to promote contacts and exchanges between the particles and the fluid. Similar questions arise from nuclear energy security, and weapons physics purposes [3,54]. Other applications cover the dynamics of biomedical sprays [3,4,30,55], environmental studies on pollutant transport [27,57,58,61,69], the formation of sandstorms,

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sediment transport, the "white water" produced by breaking waves [51], dispersion of ash during volcanic eruptions [59], powder-snow avalanches [15], etc. Such complex flows involve a wide range of length and time scales and different modeling approaches have been developed.

We focus this study to models that adopt a statistical description of the dilute phase through the distribution function $f(t,x,v)$ of the particles in phase space. Here and below, x and v are independent variables that stand for the position and velocity variables, respectively, while t represents time. In this modelling, at any position both phases can be present, and, assuming that particles are spherically shaped with typical radius $r_{\rm d}$, $\frac{4}{3}\pi r_{\rm d}^3 \int f(t,x,v)\,{\rm d}v$ defines the local volume fraction occupied by the particles. The particle distribution function obeys the collisionless Vlasov equation (or Williams equation)

$$\partial_t f + \nabla_x \cdot (vf) + \nabla_v \cdot (\mathscr{F} f) = 0. \tag{1.1}$$

(Note that, since x and v are independent variables, $\nabla_x \cdot (vf) = v \cdot \nabla_x f$.) It is coupled to a hydrodynamic system describing the evolution of the carrier phase through the drag force term $\mathscr{F}$. Denoting by $(t,x) \mapsto u(t,x)$ the velocity field of the carrier fluid, the drag force is proportional to the relative velocity

$$\mathscr{F} = D\ (u - v).$$

The coefficient D (which has the homogeneity of the inverse of a time) is given as a function of $|v-u|$, the expression of which might be quite complicated, depending on the physical characteristics of the flows [53,56]. Our ideas extend to the general case, but for the sake of simplicity, we shall restrict the description of the scheme to the simple linear case where D is a positive constant (Stokes flows). In this case, its expression is $D = \frac{9\mu}{2r_{\rm d}^2 \varrho_{\rm d}}$, where μ stands for the dynamic viscosity of the fluid, and $\varrho_{\rm d}$ the mass per unit volume of the particles (see [18] and the references therein). In the momentum equation that prescribes the evolution of the velocity u, we find a source term that accounts for the drag force exerted by the particles on the fluid:

$$S(t,x) = m_{\rm d} \int D f(t,x,v)\ (v - u(t,x))\,{\rm d}v = m_{\rm d} \int v\ \nabla_v \cdot \big(\mathscr{F} f(t,x,v)\big)\,{\rm d}v,$$

where $m_{\rm d} = \frac{4}{3}\pi r_{\rm d}^3 \varrho_{\rm d}$ stands for the mass of the particles. (For Stokes flows note that $m_{\rm d} D = 6\pi\mu r_{\rm d}$.) The difficulty for the analysis and the numerical simulation can be ranked depending on the other modelling assumptions. In particular we can distinguish the following situations:

- We can consider the carrier fluid as compressible or incompressible, inviscid or viscous. It leads to a huge variety of PDEs systems, with quite distinct features. Here, we will restrict to compressible and inviscid flows, described by standard Euler equations, with a mere state law for defining the pressure. Furthermore, we will