Practical Techniques in Ghost Fluid Method for Compressible Multi-Medium Flows

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Abstract. The modified ghost fluid method (MGFM), due to its reasonable treatment for ghost fluid state, has been shown to be robust and efficient when applied to compressible multi-medium flows. Other feasible definitions of the ghost fluid state, however, have yet to be systematically presented. By analyzing all possible wave structures and relations for a multi-medium Riemann problem, we derive all the conditions to define the ghost fluid state. Under these conditions, the solution in the real fluid region can be obtained exactly, regardless of the wave pattern in the ghost fluid region. According to the analysis herein, a practical ghost fluid method (PGFM) is proposed to simulate compressible multi-medium flows. In contrast with the MGFM where three degrees of freedom at the interface are required to define the ghost fluid state, only one degree of freedom is required in this treatment. However, when these methods proved correct in theory are used in computations for the multi-medium Riemann problem, numerical errors at the material interface may be inevitable. We show that these errors are mainly induced by the single-medium numerical scheme in essence, rather than the ghost fluid method itself. Equipped with some density-correction techniques, the PGFM is found to be able to suppress these unphysical solutions dramatically.

1 Introduction

The dynamics of compressible multi-medium flows often give rise to challenging problems in both theory and numerical simulation. The change in equation of state (EOS) is

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known to cause numerical inaccuracies or oscillations near material interfaces. In order to overcome this difficulty, various strategies have been pursued in the past two decades with an even increasing interest [1–9]. Some methods treat materials that are separated by distinct sharp interfaces by reformulating the problem using a mixture model [1–6]. An artificial EOS is usually introduced for mixture cells. This treatment, however, may fail to capture discontinuous response and result in numerical instabilities, if a shock is transmitted across an interface for instance. Comparatively speaking, an immiscible model seems to be more reasonable in the presence of sharp interfaces. Researchers can take all kinds of effective measures such as volume of fluid method [10] or more popular level set technique [11] or front tracking technique [12] to deal with it. But the interfacial state, especially when nonlinear wave interaction occurring at the interface, should be faithfully simulated to suppress any undesired numerical oscillations.

The idea of ghost fluid method (GFM), originally suggested and developed by Glimm et al. [13, 14], has provided us a simple and flexible way for handling multi-medium flows with immiscible material interfaces. The GFM-based techniques [15–19] have been improved upon and applied by many researchers to a range of problems. By specially defining the ghost fluid state, the computation can be carried out as if in a single medium. The numerical schemes for single-medium flow can be employed without any changes and the methods are easily extended to multi-dimensions. These variants in GFMs differ in the way in which the ghost fluid state is populated.

Fedkiw et al. proposed the original GFM (OGFM) [15] by using the local real fluid velocity and pressure to define the corresponding ghost fluid state. Later, the gas-water version GFM (GWGFM) [16], where the ghost fluid state is defined by employing the velocity from the water and the pressure from the gas, was specially presented for coupling non-stiff fluid (gas) and stiff fluid (water). Although the two GFMs are problem-related and not suitable for some cases like high speed jet impacting [20], the simplicity and the easy extension to multi-dimensions promote the development of these methods [6, 21–23]. In order to take into account the effects of wave interaction and material properties, Liu et al. proposed the modified GFM (MGFM) [17] by carrying out characteristic analysis on the waves arriving at the interface and solving the local Riemann problem. Following the idea of Riemann-problem-based technique, the interface-interaction GFM (IGFM) [18] and the real GFM (RGFM) [19] have also been developed recently. The Riemann-problem-based algorithm, discussed in this paper, is characterized by (approximately) solving a multi-medium Riemann problem to define the ghost fluid state. This differs from the OGFM and the GWGFM where the ghost fluid states are defined via using the local flow state or extrapolating from the real fluid. These Riemann-problem-based techniques have been shown to be robust and less problem-related and successfully applied to solve a multitude of problems involving strong shocks interacting with gas-gas, gas-water interfaces and even fluid-structure coupling problems [17–20, 24–28]. Furthermore, it has been proved that the error estimate by the MGFM is "third-order accurate" in the vicinity of the interface for a multi-medium Riemann problem [29, 30].

Besides the MGFM where the ghost fluid state is defined by using the interfacial state,