Commun. Comput. Phys.
doi: 10.4208/cicp.200114.090316a

Vol. **20**, No. 3, pp. 811-834
September 2016

A Contact Line Dynamic Model for a Conducting Water Drop on an Electrowetting Device

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Communicated by Ming-Chih Lai

Received 20 January 2014; Accepted (in revised version) 9 March 2016

Abstract. The static shape of drop under electrowetting actuation is well studied and recent electrowetting theory and experiments confirm that the local contact angle (microscopic angle) is unaffected while the apparent contact angle (macroscopic angle) is characterized by the Lippmann-Young equation. On the other hand, the evolution of the drop motion under electrowetting actuation has received less attention. In this paper, we investigate the motion of a conducting water drop on an electrowetting device (EWD) using the level set method. We derive a contact line two-phase flow model under electrowetting actuation using energy dissipation by generalizing an existing contact line model without the electric field. Our model is consistent with the static electrowetting theory as the dynamic contact angle satisfies the static Young's equation under equilibrium conditions. Our steady state results show that the apparent contact angle predicted by our model satisfies the Lippmann-Young's relation. Our numerical results based on the drop maximum deformation agree with experimental observations and static electrowetting theory. Finally, we show that for drop motion our results are not as good due to the difficulty of computing singular electric field accurately. Nonetheless, they provide useful insights and a meaningful first step towards the understanding of the drop dynamics under electrowetting actuation.

AMS subject classifications: 76T99, 76W05, 65M06

Key words: Moving contact line, contact angle, level set method, electrowetting, slip conditions.

1 Introduction

Electrowetting devices (EWD) are gaining popularity in digital microfluidics as they can be used to create, transport, break up and merge drops [1–3] and transport heat [4].

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Electrowetting as a popular technique to manipulate droplets is introduced about two decades ago. Studies have been carried out to understand the steady-state droplet shape response to an external electric field. On the other hand, many microfluidic applications will benefit from a better understanding the dynamics of droplets under electrowetting actuation [5,6].

Contact line dynamics without electric field have been extensively studied both experimentally and theoretically, cf. [7] and references therein. Several computational methods have also been proposed for solving moving contact line problems without electric field, such as volume of fluid method [8–10], front tracking method [11], and level set method [12–15]. However, there is much less published work on contact line dynamics under the actuation of electric field, which was considered as one of the open problems in electrowetting [16]. In this paper, we present a dynamic contact line model under the actuation of electric field. We derive our moving contact line model from the energy dissipation point of view, by generalizing the approaches for the non-electric case [17].

A conceptual EWD set up is shown in Fig. 1, where a conducting drop surrounded by a dielectric fluid is placed on a hydrophobic dielectric layer, with an applied electric field between the drop and electrode under the dielectric layer. In the early electrowetting literature [1, 2], it was widely accepted that the equilibrium interface of drop under electrowetting is a sphere shape, and the contact angle under electrowetting follows Lippmann-Young's relation [18]

$$\cos(\theta_{\rm B}) = \cos(\theta_0) + {\rm B}, \tag{1.1}$$

where ${\rm B} = \varepsilon_d \varepsilon_0 V^2 (2\gamma d)^{-1}$ is the electric Bond number, d is the height of the insulating dielectric layer, V is the applied voltage, θ_0 is the Young's angle which is the equilibrium contact angle without electric field, $\theta_{\rm B}$ is the equilibrium contact angle, γ is the interfacial tension between the two fluids, ε_0 is the vacuum permittivity, ε_d is the dielectric constant of the dielectric layer. However, recent experiments and static electrowetting theory [19,20] show that the equilibrium interface of the drop under electrowetting takes up a non-spherical shape with an interface boundary layer near the contact lines. The Lippmann-Young's relation is only applicable to the apparent static contact angle

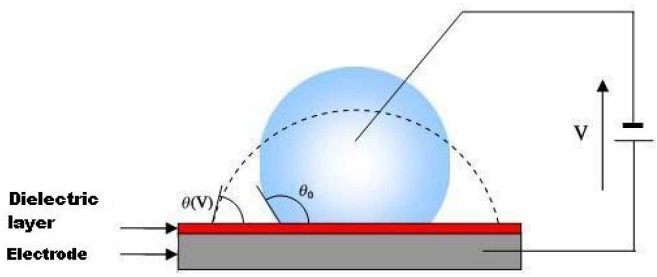

Figure 1: Electrowetting device.