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Influence of Finite Size Effects on the Fulde-Ferrell-Larkin-Ovchinnikov State

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Abstract. The Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state is the superconducting phase for which the Cooper pairs have a non-zero total momentum, depending on the splitting of the Fermi surface sheets for electrons with opposite spin. In infinite systems the momentum is a continuous function of the temperature. In this paper, we have shown how the finite size of the system, through the discretized geometry of the Fermi surface, affects the physical properties of the FFLO state by introducing discontinuities in the Cooper pair momentum. Our calculation in an isotropic system show that the superconducting state with two opposite Cooper pair momenta is more stable than state with one momentum also in nano-size systems, where finite size effects play a crucial role.

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Key words: Superconductivity, FFLO, finite size, numerical instability.

1 Introduction

The *classical* theory of superconductivity, proposed by Bardeen, Cooper and Schrieffer (BCS) in 1957 [1,2], explains this specific state using the concept of *Cooper pairs*. Namely, spin-singlets are formed by an electron in state $(\boldsymbol{k}\uparrow)$ and a second one in state $(-\boldsymbol{k}\downarrow)$. Thus, we can speak about *stationary* Cooper pairs, because the total momentum of such pair equals zero. However, in 1964 two groups, Fulde and Ferrell [3], and Larkin and Ovchinnikov [4], independently proposed a state with *non-stationary* (moving) Cooper

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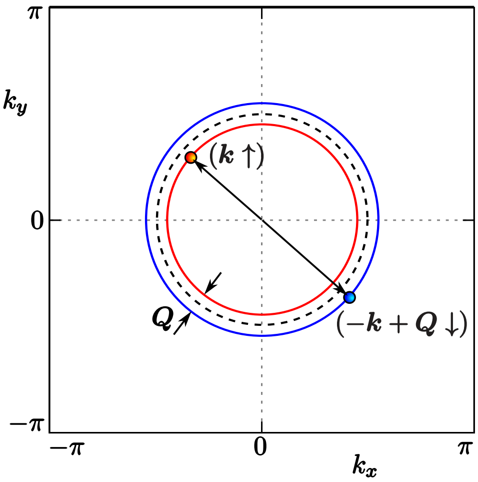

Figure 1: (Color on-line) Schematic presentation of the formation of the Cooper pairs in the BCS and FFLO states. Circles represent the Fermi surfaces in the absence (dashed line) and in presence (solid lines) of an external magnetic field. In the BCS case, every electron in the $(\boldsymbol{k},\uparrow)$ state is paired with an electron in the $(-\boldsymbol{k},\downarrow)$ state, since the Fermi surfaces for electrons with opposite spins are the same. For non-zero magnetic field, the Fermi surface sheets for electrons with spin $\uparrow$ (red line) and spin $\downarrow$ (blue line) are shifted by the Zeeman effect. In this case, the FFLO state can be formed, because electrons in the $(\boldsymbol{k},\uparrow)$ state pair with an electron in $(-\boldsymbol{k}+\boldsymbol{Q},\downarrow)$ state.

pairs, characterized by a finite total momentum $\boldsymbol{Q}$. This state is known as the FFLO phase, which can be realized at low temperature and high magnetic fields when Cooper pairs can be formed by electrons with opposite spins $(\boldsymbol{k}\uparrow,-\boldsymbol{k}+\boldsymbol{Q}\downarrow)$ between the spin-split sheets of the Fermi surface. The mechanism for the Cooper-pair formation with finite momentum is illustrated in Fig. 1.

We should be aware that the magnetic field can destroy superconductivity in two distinct ways, by either orbital or paramagnetic effects [5]. The orbital effect is connected with the rise of the Abrikosov vortex state, while the paramagnetic one is associated with the Zeeman effect. For superconductors limited by the latter, the upper critical magnetic field is called the Pauli (or Clogston-Chandrasekhar) limit [6,7]. This effect is connected with the splitting of the Fermi surface for electrons with opposite spin. It has an important influence on the FFLO phase formation as well as on phase separation [8,9].

Since the proposal of the FFLO phase, many groups have sought materials where it can be physically realized. As written above, the upper critical magnetic field of potential candidates should be given by the Pauli limit. Among such materials are (*quasi* two-dimensional) organic superconductors [10], heavy fermions systems [11–13] and iron-based superconductors (see Refs. [14–19] and references cited therein). Moreover, the FFLO phase can be also realized in systems where a shift of the Fermi surface sheets for electrons with opposite spin exists, as in ultra-cold fermion gases [20–22] or dense quark matter [23,24], as well as in solid state in case of inter-band pairing [25] or nanofilms [26–28].