Commun. Comput. Phys.
doi: 10.4208/cicp.220415.080816a

Vol. **21**, No. 3, pp. 782-807
March 2017

Solving Vlasov-Poisson-Fokker-Planck Equations using NRxx method

Yanli Wang[1,*] and Shudao Zhang[1]

[1] *Institute of Applied Physics and Computational Mathematics, Beijing 100094, P.R. China.*

Received 22 April 2015; Accepted (in revised version) 8 August 2016

Abstract. We present a numerical method to solve the Vlasov-Poisson-Fokker-Planck (VPFP) system using the NRxx method proposed in [4, 7, 9]. A globally hyperbolic moment system similar to that in [23] is derived. In this system, the Fokker-Planck (FP) operator term is reduced into the linear combination of the moment coefficients, which can be solved analytically under proper truncation. The non-splitting method, which can keep mass conservation and the balance law of the total momentum, is used to solve the whole system. A numerical problem for the VPFP system with an analytic solution is presented to indicate the spectral convergence with the moment number and the linear convergence with the grid size. Two more numerical experiments are tested to demonstrate the stability and accuracy of the NRxx method when applied to the VPFP system.

AMS subject classifications: 35Q83, 65L20, 78M05

Key words: Moment closure, Fokker-Planck operator, VPFP system, spectral convergence.

1 Introduction

The Vlasov-Poisson-Fokker-Planck (VPFP) system describes the dynamics of the charged particles which are subject to the electrostatic force coming from their Coulomb interaction. It is obtained by adding a diffusion term (FP operator term) onto the Vlasov-Poisson (VP) equations. Though the stochastic modification is simple, the FP operator term greatly changes the properties of the VP equations.

The existence and uniqueness of the weak and classical solutions to the VPFP and related systems have been well studied. In [26], the uniqueness and stability of the weak solution to the VFP equation were proved by R. Rautmann. H. Neunzert in [24] generalized the results concerned with the existence and uniqueness of the solutions from

*Corresponding author. *Email addresses:* wang_yanli@iapcm.ac.cn (Y. Wang), zhang_shudao@iapcm.ac.cn (S. Zhang)

the ordinary Vlasov equation to the VFP system using a probabilistic method. The fully deterministic proof of the existence of the global solutions to the VFP system in one and two space dimensions were proposed in [14], which also proved that the solutions to the VPFP system converged to those to the VP equations, when the coefficients in the FP operator term went to zero. The global existence of the solution to the VPFP system with small initial data was raised in [30]. Later on, F. Bouchut in [2,3] proved the existence and uniqueness of the smooth solution to the VPFP system in three dimensions. For the property of VPFP system, some other models are derived based on it. The high-field model of the VPFP system was studied in [17]. Moreover, the parabolic limit model of the VPFP system was discussed in [25].

Due to the complex phenomena in the plasma, the numerical simulation plays an important role in the study of the VPFP and related equations. For the stochastic properties of the FP operator, the random particle method is one of the most effective methods. An analysis of the method was raised in [18] and a computational study of it was carried out in [1, 27]. The finite difference scheme was also proposed for the periodic 1D VPFP system in [13, 28]. The deterministic methods, which approximated the solution along the characteristic curves associated with the transport part of the VPFP system were also proposed, eg. [19,27,28]. Further more, S. Wollman in [31,32] combined the deterministic particle type computation with a process of periodically reconstructing the distribution function on a fixed grid. This method was extended to the two dimension case and the Vlasov-Maxwell-Fokker-Planck system as well. In recent years, some asymptotic preserving schemes were proposed to solve the VPFP system under different field regions. For example, in the high-field regime, the forcing term containing the electric potential is stiff, and an explicit method would require $\Delta t \sim \min(\Delta x, \epsilon\Delta v)$, where $1/\epsilon$ is the stiff coefficient of the forcing term. Moreover, the diffusive nature of the Fokker-Planck operator poses the constrain $\Delta t \sim \mathcal{O}(\epsilon\Delta v^2)$. When ϵ is small, the numerical computation becomes quite expensive. Shu in [12] performed some numerical simulations on the high-field model. And the numerical method proposed in [22] could capture the high-field limit of the VPFP system using large time steps and coarse meshes. Besides, the spectral method was also used to solve the Fokker-Planck-Landau system in [15]. Recently, an approach based on the moment method was proposed in [29], and therein the distribution function was expanded using the Hermite polynomials with a prescribed macroscopic velocity chosen as the expansion center and a prescribed temperature of the particles as the scaling factor.

In the past years, a regularized moment method (the NRxx method) was developed in [4, 7, 9] to numerically solve the Boltzmann equation. The NRxx method adopts the Hermite polynomial expansion to approximate the distribution functions, with the basis function shifted by the local macroscopic velocity and scaled by the square root of the local temperature. The new regularized model proposed in [4, 5] guaranteed the global hyperbolicity of the regularized moment system. Due to the local well-posedness provided by the global hyperbolicity, it was eventually accessible that approximating the distribution functions far away from the equilibrium distribution by the stable simula-