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Comprehensive Studies on Rarefied Jet and Jet Impingement Flows with Gaskinetic Methods

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Abstract. This paper presents comprehensive studies on two closely related problems of high speed collisionless gaseous jet from a circular exit and impinging on an inclined rectangular flat plate, where the plate surface can be diffuse or specular reflective. Gaskinetic theories are adopted to study the problems, and several crucial geometry-location and velocity-direction relations are used. The final complete results include flowfield properties such as density, velocity components, temperature and pressure, and impingement surface properties such as coefficients of pressure, shear stress and heat flux. Also included are the averaged coefficients for pressure, friction, heat flux, moment over the whole plate, and the averaged distance from the moment center to the plate center. The final results include complex but accurate integrations involving the geometry and specific speed ratios, inclination angle, and the temperature ratio. Several numerical simulations with the direct simulation Monte Carlo method validate these analytical results, and the results are essentially identical. Exponential, trigonometric, and error functions are embedded in the solutions. The results illustrate that the past simple cosine function approach is rather crude, and should be used cautiously. The gaskinetic method and processes are heuristic and can be used to investigate other external high Knudsen number impingement flow problems, including the flowfield and surface properties for high Knudsen number jet from an exit and flat plate of arbitrary shapes. The results are expected to find many engineering applications.

AMS subject classifications: 82C40, 76P05, 65C05
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1 Introduction

Gaseous jets expanding into a vacuum and jet impingement on a plate are two fundamental fluid dynamic problems with many reports in the literature. As the counterpart

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to the continuum flow situation, highly rarefied jet and jet impingement flows provide bounding limits and insights to many problems by solely including molecular movements. In many applications involving high Knudsen (Kn) numbers or high speed velocities, the contributions from particle collisions are insignificant. One important example is atomic/molecular beams [1,2] which lead to many important discoveries. Other nontrivial applications include materials processing inside vacuum chambers [3] and rocket plume effects [4,5].

Because there are so many applications related with the rarefied jet and jet impingement, it is challenging to provide a detailed review. Here we only provide several examples for rocket plume, which is crucial to the rocket and spacecraft community. As the most important signature, rocket plume is a key component for space propulsion, inferred radar detections and performance evaluations. Due to this importance, many communities have been investigating rarefied gaseous jet/plume impingement flows for decades. For example, the biennial international rarefied gasdynamics symposiums usually collect papers on gaseous jet/jet impingement and molecular beams; the NASA Johnson Space Center developed a versatile particle simulation package to simulate plume impingement [6] on spacecraft surface. Kannenberg and Boyd [7]used the cosine law and the Simons model [8] and particle simulation method to compute the density of a plume impinging on a flat plate. Ivanov [9] and his colleagues performed particle simulations of plume flows from a nozzle.

Compressible flows usually can be divided into four categories according to the Knudsen number (Kn) [10]:

$$Kn=\lambda/L, \tag{1.1}$$

where λ is the molecular mean free path, and L is a characteristic length. These four regimes are: continuum ($0<Kn<0.01$), velocity slip and temperature jump ($0.01<Kn<0.1$), transitional ($0.1<Kn<10$), and free molecular or collisionless ($Kn>10$). For jet and jet impingement on a flat plate, there are abundant studies concentrating in the continuum regime, and many numerical studies at the transition regime; at the collisionless flow regime, there are numerical and experimental studies on the flowfield and surface properties. For high speed, gaseous, collisionless flows out of an exit, many past studies adopted several simplifications. For example, Noller [11] proposed a solid angle treatment to implicitly consider the nozzle exit geometry and obtained the plume density field expressed with integrations over solid angles which are subtended by a flowfield point and many small surface elements; the cosine law/Simons model [8] treats a rocket plume as from a point source; Narasimha's early investigation [12] indicated that the plume solution is rather complicated with many cosine functions. Another rocket plume treatment, which is also based on collisionless flows, was suggested by Woronowicz [13]. His treatment splits the exit into many small segments, as such, the density and pressure distributions in the flowfield can be computed numerically. Further, he proposed the concept of starting surfaces alleviating the difficulty of this problem.

This paper presents recent work on seeking exact solutions to the problems of colli-