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Application of High Dimensional B-Spline Interpolation in Solving the Gyro-Kinetic Vlasov Equation Based on Semi-Lagrangian Method

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Abstract. The computation efficiency of high dimensional (3D and 4D) B-spline interpolation, constructed by classical tensor product method, is improved greatly by precomputing the B-spline function. This is due to the character of NLT code, i.e. only the linearised characteristics are needed so that the unperturbed orbit as well as values of the B-spline function at interpolation points can be precomputed at the beginning of the simulation. By integrating this fixed point interpolation algorithm into NLT code, the high dimensional gyro-kinetic Vlasov equation can be solved directly without operator splitting method which is applied in conventional semi-Lagrangian codes. In the Rosenbluth-Hinton test, NLT runs a few times faster for Vlasov solver part and converges at about one order larger time step than conventional splitting code.

AMS subject classifications: 65D05, 65Z05, 68U20, 82D10

Key words: High dimensional interpolation, semi-Lagrangian, gyro-kinetic simulation, numerical Lie transform.

1 Background

In magnetic fusion plasmas or laser-plasmas, it is important to understand the dynamics of charged particles. In kinetic theory, the plasma is described by the distribution function $f(\boldsymbol{x},\boldsymbol{v},t)$ in the six dimensional phase space $(\boldsymbol{x},\boldsymbol{v})$, with $\boldsymbol{x},\boldsymbol{v}$ the position and velocity of the particle. Due to the strong toroidal magnetic field in a tokamak, the phase space can be simplified to five dimensional phase space by using gyro-center coordinates $Z = Z(\boldsymbol{X},v_{\parallel},\mu)$, with $\boldsymbol{X}$ the position of gyro-center, $v_{\parallel}$ the parallel velocity and μ the magnetic

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moment. In the absence of collisions, the particle distribution satisfies the gyro-kinetic Vlasov equation,

$$\frac{dF}{dt}=\frac{\partial F}{\partial t}+\dot{\boldsymbol{X}}\frac{\partial F}{\partial \boldsymbol{X}}+\dot{v}_{\|}\frac{\partial F}{\partial v_{\|}}=0. \tag{1.1}$$

Here, the total time derivative $\frac{d}{dt}$ in Eq. (1.1) is taken along the gyro-center orbit in phase space, which indicates that the distribution function F is a constant along its characteristics. The electromagnetic fields are computed self-consistently through the Maxwell or Poisson equations.

There are three methods to solve the Vlasov equation, the PIC (Particle-In-Cell), the Eulerian, and the semi-Lagrangian methods. The PIC method [1–3] is based on a Lagrangian description. The plasma is described by a statistical sampling of macro-particles. Their trajectories are followed in time with electromagnetic fields computed on mesh nodes. In order to get the sources for the computation of the electromagnetic field, the macro-particles are deposited into mesh nodes. The PIC method has a good conservative property, but suffers from sampling noise. The Eulerian method [4–6], which discretizes the Vlasov equation on mesh of the phase space, can offer a good alternative to the PIC method. This method is usually computationally more demanding than the PIC method and has the CFL restriction on the time step, while it does not suffer from the statistical sampling noise faced by the PIC method. The semi-Lagrangian method [7–9] takes advantage of both the Lagrangian and the Eulerian approaches. It allows a relatively accurate description of the phase space through using a fixed mesh and avoids the traditional step size restriction through using the invariance of the distribution function along the trajectories. In standard semi-Lagrangian method, particles at mesh nodes are advanced backward in time. The distribution function $F(t^{n+1})$ at mesh nodes is interpolated by $F(t^n)$ at previous time step. In order to avoid the high dimensional interpolation, the high dimensional Eq. (1.1) is transformed into a few 1D or 2D equations by operator splitting [10, 11]; an example of this operator splitting semi-Lagrangian method is the GYSELA code [8, 9]. However, the operator splitting can't recover Vlasov equation completely and brings error. This will introduce artificial dissipation to the semi-Lagrangian simulation. There is also a forward semi-Lagrangian method which avoids operator splitting. The characteristic curves are advanced in time and a deposition procedure on the phase space grid, similar to the procedure used in PIC methods for the configuration space only, is used to update the distribution function. It enables the use of large time steps and recovers a good global behavior, but it is less accurate than the classical backward semi-Lagrangian method [12].

The interpolation algorithm in these three methods are different. In the PIC and the forward semi-Lagrangian methods, data at regular mesh nodes are interpolated from data at irregular points through the usual deposition technique ("scattering" procedure), is shown in Fig. 1(b). In the backward semi-Lagrangian method, data at irregular points are interpolated from data at regular mesh nodes through using the spline interpolation method, is shown in Fig. 1(a). However, only 1D and 2D spline interpolation are widely