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An XFEM Implementation of a Projection Method for 3D Incompressible Two-Fluid Flows with Arbitrary High Contrasts in Material Properties

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Abstract. This paper presents an *XFEM* implementation of a projection algorithm to compute in an Eulerian framework 3D incompressible two-fluid flows with arbitrary high contrasts in material properties. It is designed to deal with both strong and weak discontinuities across the interface for pressure and velocity fields, respectively. A classical enrichment function accounts for velocity gradient discontinuities across the interface and a new quadratic enrichment function accounts for pressure discontinuities across the interface. A splitting of two-fluid elements is performed to achieve accurate numerical integrations, meanwhile a scaling coefficient accounting for both physical and geometrical considerations alleviates ill-conditioning. Various validations have been carried and very good solution accuracy is achieved even on coarse meshes, as from the minimal mesh not conforming to the interface. This implementation enables to compute accurate solutions regardless of discontinuity magnitude (arbitrary high contrast in material properties) and mesh size of two-fluid elements, which can constitute a decisive advantage for large size 3D computations.

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Key words: Incompressible two-fluid flows, strong and weak field discontinuities, Extended Finite Element Method, projection algorithms.

1 Introduction

Two-fluid flows involve two non-miscible fluids, which are therefore separated by a sharp interface, whose location is usually part of the problem unknowns to be determined. They take place in a large collection of fluid mechanics problems ranging from environmental and geophysical to industrial processing situations. Many numerical methods have been developed to deal with such problems, but their accuracy and computational efficiency mainly depend on their capabilities to deal with two key-points: i) the

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topological complexity of the interface(s) and its (their) potential changes in the course of time; ii) the material discontinuities across the interface(s) and their related velocity and pressure field discontinuities.

Among many others, the Extended Finite Element Method (*XFEM*) is well suited to account for material discontinuities in an Eulerian framework. Indeed, it has emerged from Element-Free Galerkin Methods and inherited many of their techniques [1]. It is designed to approximate discontinuous fields by supplementing the classical continuous polynomial approximations with extra discontinuous ones related to the known physical discontinuities. Four main features are meaningful to the computational efficiency of an *XFEM* implementation: (i) the interface representation, (ii) the chosen enrichment functions for strong and weak discontinuities, (iii) the numerical integration of weak integral forms in elements crossed by the interface and (iv) the way to reduce ill-conditioning that arises from localized tiny supports of enrichment functions.

In the framework of two-fluid flows, the pioneering work of Chessa and Belytschko [2] acknowledged the Level Set Method [3–5] to be a very convenient way to represent the interface geometry in *XFEM*, whatever it is steady or time-dependent. Based on this statement most enrichment functions are nowadays based on the Level Set function, defined as the signed distance to the interface [2]. Optimal convergence rates have been reported for strong pressure discontinuity problems (related to surface tension) with the *sign* enrichment function [6], meanwhile only sub-optimal convergence rates are experienced for continuous fields with discontinuous gradients (related to different densities and/or dynamical viscosities) with the *abs* enrichment function [7, 8]. The cure for the latter was to supplement blending elements (that have both enriched and non-enriched nodes) with a smoothed ridge enrichment function [7] or a linearly decreasing weight function [9], so that their enrichment functions are shifted to vanish at edge nodes [10,11]. As an Eulerian approach the mesh does not coincide with the interface, so discontinuities in material properties lead to discontinuous integrands in crossed elements and Gauss quadratures no longer produce accurate results. To overcome this problem one splits crossed elements into homogeneous sub-domains on each side of the interface, so that integrands remain piecewise continuous [6]. In three dimensional problems such splittings are sometimes computationally cumbersome and tricky, resulting in either two-stage procedures [6,12] or one-stage ones [13]. Such a splitting strategy into sub-domains with curved, higher-order edges [14] or faces [15] has recently pushed a step forward enabling to perform accurate numerical integrations with reduced numbers of quadrature points.

A recurrent *XFEM* issue is the potentially devastating ill-conditioning that arises in algebraic systems resulting from cut elements with large ratios of volumes on both sides of the interface [12]. Such ill-conditioning not only critically degrades the convergence rate of iterative solvers, but can also in extreme cases lead to completely erroneous results even with direct solvers. Several cures have been proposed, either changing the enrichment function itself [16] or its nodal support, discarding enriched degrees of freedom associated with too tiny supports [17] or moving them to recover a better condition number