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Development of Finite Element Field Solver in Gyrokinetic Toroidal Code

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Abstract. A new finite element (FE) field solver has been implemented in the gyrokinetic toroidal code (GTC) in attempt to extend the simulation domain to magnetic axis and beyond the last closed flux surface, which will enhance the capability the GTC code since the original finite difference (FD) solver will lose its capability in such circumstances. A method of manufactured solution is employed in the unit fidelity test for the new FE field solver, which is then further verified through integrated tests with three typical physical cases for the comparison between the new FE field solver and the original finite difference field solver. The results by the newly implemented FE field solver are in great accord with the original solver.

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1 Introduction

In the research of plasma physics, simulations have always served as an effective tool due to the complexity of theoretical analysis and the high cost of experiments. After

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*Corresponding author. *Email addresses:* `wzhang@iphy.ac.cn` (W. Zhang), `tracyf@mail.ustc.edu.cn` (H. Feng), `zhihongl@uci.edu` (Z. Lin), `xiaohe@iscas.ac.cn` (X. Zhufu), `xu_jin@iscas.ac.cn` (J. Xu), `jtcao@iphy.ac.cn` (J. Cao), `dli@cashq.ac.cn` (D. Li) several decades of fast development in the capability of high-performance-computing, it has become feasible to conduct massively parallel simulations to investigate more meaningful and complex physics processes using equilibrium and profiles close to realistic discharges in fusion plasmas. Along with the progress in computing power, a set of gyrokinetic theory [1–11] have been proposed and established to construct a set of simple theoretical and numerical models by eliminating the fine-scale gyro-phase dependence through gyroaveraging, which reduces the original phase space dimensionality from six to five. This not only assists the profound comprehension of low frequency physics, such as the anomalous transport that is critical for the magnetic fusion, but also facilitates the development and application of massively parallel simulation codes.

As one of the existing well benchmarked gyrokinetic codes, the gyrokinetic toroidal code (GTC) [10,12] is built upon the first-principles and adopts an efficient low-noise perturbative δf simulation method. The particle-in-cell (PIC) scheme is also utilized so that particles are treated with a Lagrangian scheme while fluid moments and field information are calculated with an Eulerian scheme. The capability of GTC has been extensively expanded and verified to deal with a wide range of physical problems, which include neoclassical and turbulence transport [13,14], energetic particle transport by microturbulence [15, 16], Alfven eigenmodes [17–20], radio frequency heating [21], static magnetic island [22] and current-driven instabilities [23,24].

GTC employs the magnetic flux coordinate system (ψ,θ,ζ) [25], where ψ is the poloidal magnetic flux, θ is the poloidal angle and ζ is the toroidal angle. The introduction of such a system makes it convenient to decompose a vector into components parallel and perpendicular to the direction of the magnetic field, and to separate the rapid particle motion along the magnetic field lines from the slow motion across the lines, which promotes the simplicity in theory analysis and efficiency in numerical simulation. Originally, GTC focused on the physics of microturbulence and the transport in the core area that are generally located in between a belt region away from the magnetic axis and Scrape-Off layer (SOL). When global modes and instabilities, such as kink mode and tearing mode, are considered, the on-axis region cannot be ignored and starts playing an important role. Meanwhile, the physics in the SOL region, a subregion of the halo, is the key for a holistic understanding to the plasma exhaust problem to ensure that the material surfaces will survive harsh plasma conditions and not interfere with core plasma in a magnetic fusion reactor. In order to extend the physics capabilities to the on-axis and SOL regions while improving numerical properties and avoiding the singularities naturally held by the concentric curvilinear coordinate systems, the finite element (FE) method is introduced and implemented in GTC [26] to replace the current on-axis solution which is simply an extrapolation of the gyrokinetic Poisson equation solution [23] to the magnetic axis.

The outline of this paper is as follows. In Section 2, equilibrium data setting and the particle-field interaction loop are introduced together with the related computing meshes. In Section 3, a brief introduction to the original finite difference (FD) Poisson solver is presented and the discrete form of the FD Poisson equation is demonstrated. In Section 4, the implementation of the finite element solver is introduced. In Section 5, the