Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2017-0162

Vol. **24**, No. 3, pp. 791-809
September 2018

Incorporation of NURBS Boundary Representation with an Unstructured Finite Volume Approximation

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Communicated by Kun Xu

Received 3 August 2017; Accepted (in revised version) 11 October 2017

Abstract. For compressible flow computations, the present paper extends the ETAU (enhanced time-accurate upwind) unstructured finite volume (FV) scheme to handle curved domain boundary with better accuracy. For the interior cells in the computational domain or the boundary cells with straight line boundary, the original ETAU scheme with second order accuracy in space and time is applied. For those boundary cells with the curved geometry, a more accurate Non-Uniform Rational B-Spline (NURBS) representation of the boundary is considered. The NURBS is commonly employed in computer aided design (CAD) to construct complex geometries. Here, it yields an exact geometry expression of complex boundary geometry. By combining ETAU with NURBS, the NURBS incorporated ETAU scheme (NETAU) is proposed for more accurate geometrical representation and fluxes evaluation. Details of the computing procedure of the geometry and surface fluxes for cells on the curved boundary, such as special transformation strategies and merging of ETAU and NURBS, are introduced and implemented. With NURBS, the NETAU scheme are geometrically versatile and more flexible. Several two-dimensional (2D) numerical cases are investigated to demonstrate the performance, computing efficiency and benefits of the NETAU scheme. The numerical results show that, for flows with low speed and high Reynolds number, the NETAU scheme provides more accurate pressure distribution on curved boundary than the original ETAU scheme. Meanwhile, the high-speed flow case shows that the NETAU scheme is still stable for high Mach number problem with shocks. Thus, the NETAU scheme potentially provides an accurate tool to describe complex geometry in computational fluid dynamics (CFD) simulations. It will help to reduce computational costs and enhances accuracy for flow domain dominated by complex geometries, with features such as high curvature and sharp edges.

AMS subject classifications: 65M08, 65M12, 65M55

Key words: NURBS, finite volume scheme, curved boundary, unstructured grid.

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1 Introduction

In computational fluid dynamics (CFD), finite volume (FV) method are popular and commonly applied for industrial applications, primarily due to their robustness and flexibility for complex geometries. [1–3] The FV method use the integral form of the conservation equations. Then, the Gauss divergence theorem is applied to a control volume (CV). The implementation of FV schemes usually consists of two steps [4, 5]. In the first (reconstruction) step, with given initial conditions, the cell average-flow variables are reconstructed into linear or higher-order polynomials within the CV. The second (evolution) step involves computing the surface fluxes of the CV; the cell averaged values of flow variables are then obtained at the next time level. For upwind schemes, including Godunov scheme [6], TVD scheme [7], the essentially non-oscillatory (ENO) scheme [8], weighted essentially non-oscillatory (WENO) schemes [9], discontinuous Galerkin (DG) scheme [10] and so on, the surface fluxes are calculated by using a Riemann solver [11].

For the applications of FV schemes, the practical geometries are generally complex, e.g. turbine machinery and airfoils. The importance of boundary geometry description is obvious for accurate simulation in CFD applications. The reality is that, up until now, the linear approximation for complex geometries [12] is still the most common strategy in the generation of structured and unstructured grids, especially in the commercial software. In fact, early in 1969, Moretti [13] has demonstrated the inaccuracy of linear approximation for curved geometries. The linear meshing approximations of curved boundaries may cause mandatory errors for the complex geometry unless further refining the grids. However, with extremely refined grids, the requirement of CPU calculation time will be excessively increased.

In general, more accurate treatment is taking into consideration of the curved geometry instead of linear approximation. Dating back to 1978, Rizzi [14] took into account the curvature of boundary instead of treating it locally as straight lines. Rizzi's formula is streamline differentiating the normal velocity equation, where the normal is taken relative to the curved physical geometry. Dadone [15, 16] extended Rizzi's formula and obtained density and tangential velocity from the constant entropy and total-enthalpy vortex model. Dadnone's results showed the influence of accurate geometrical model in the numerical solution of Euler equations in the FV approximation. Wang and Sun [17] extended Dadone's method to unstructured grids and arbitrary curved boundaries for Euler equations. Then, Catalano [18] extended this method to a local refinement multigrid technique in Cartesian grid solver for inviscid subsonic flow. Landmann et al. [19] have demonstrated the importance of accurate boundary description in numerical simulation of viscous flows with curved geometry, using cubic function to approximate curved boundary. Bassi and Rebay [20] showed that high order schemes (e.g. DG schemes) are highly sensitive to boundary description accuracy and concluded that high-order approximation of geometry is crucial for accurate simulation. Krivodonova and Berger [21] proposed a method to approximate the curved geometry by an arc of circle passing through the integration points. The radius of the circle is the average of two