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Lipschitz and Total-Variational Regularization for Blind Deconvolution

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Abstract. In [3], Chan and Wong proposed to use total variational regularization for both images and point spread functions in blind deconvolution. Their experimental results show that the detail of the restored images cannot be recovered. In this paper, we consider images in Lipschitz spaces, and propose to use Lipschitz regularization for images and total variational regularization for point spread functions in blind deconvolution. Our experimental results show that such combination of Lipschitz and total variational regularization methods can recover both images and point spread functions quite well.

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1 Introduction

It is well-known that recovering both an image u and a point spread function (PSF) k is a mathematically ill-posed problem. This is called a blind deconvolution problem. In the literature, there are many methods for simultaneously recovering both u and k, see for instance [3, 6, 8, 9, 12, 13]. In [3], Chan and Wong proposed to use total variational (TV) regularization for both images and PSFs in blind deconvolution. The motivation for using TV regularization for the PSF is due to the fact that some PSFs can have edges, see [3]. They find u and k by minimizing the cost function defined as follows:

$$\min_{(u,k)} E(u,k) = \min_{(u,k)} \{ ||k*u-f||_2^2 + \alpha_1 \int |\bigtriangledown u| + \alpha_2 \int |\bigtriangledown k| \}. \tag{1.1}$$

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Here $*$ denotes convolution operator, u and k are the original image and the exact blur respectively, f is the observed image, α_1 and α_2 are positive parameters which measure the trade off between a good fit and the regularity of the solutions u and k. Chan and Wong devised fast numerical algorithms for solving the minimization problem (1.1). Their algorithm can recover both the image and PSF without any a priori information on the PSF. However, their experimental results show that the detail of the restored images cannot be recovered. The main reason is the TV regularization is used for the image.

In this paper, we consider images in Lipschitz spaces where a wide class of nonsmooth images can be modeled, see for instance [1,4,5], and propose to use Lipschitz regularization for u and total variational regularization for k in blind deconvolution. Similar to (1.1), we formulate the blind deconvolution problem as follows:

$$\min_{(u,k)} E(u,k) = \min_{(u,k)} \{ ||k*u-f||_2^2 + \alpha_1 (||u||_2^2 + \gamma ||z_t * u||_2^2) + \alpha_2 \int |\bigtriangledown k| \}, \tag{1.2}$$

where α_1 and α_2 are the regularization parameters, and the image regularization term based on the Wiener filtering is given by $(||u||_2^2 + \gamma ||z_t * u||_2^2)$, see [1] for detail. Here z_t $(t>0)$ is a function related to the Poisson kernel used to calibrate the lack of smoothness of u at the scale t with the weighting $\gamma > 0$. The construction of image regularization term is based on the assumption that the images belong to proper Lipschitz spaces.

The outline of this paper is as follows. In Section 2, we will introduce Lipschitz regularization method and consider alternating minimization algorithm for solving (1.2). In Section 3, numerical results will be presented. Our experimental results show that such combination of Lipschitz and total variational regularization methods can recover both images and point spread functions quite well. The detail of the restored images can be recovered. Finally, the concluding remarks are given in Section 4.

2 Blind deconvolution by Lipschitz regularization

Total variational regularization method can efficiently recover edges of images, it loses much fine scales of the images due to the assumption that images are represented by bounded variation functions [7]. In this paper, we consider images in Lipschitz spaces. In order to represent images in Lipschitz spaces, the Poisson singular integral is used to measure the smoothness of an image. The details can be found in [1].

The Poisson singular integral operator is defined as a linear operator on $\mathbf{L}^2(\mathbf{R}^2)$ as follows:

$$\Psi_t u(x,y) = \int_{\mathbf{R}^2} \psi_t(w,v) u(x-w,y-v) dw dv. \tag{2.1}$$

For fixed $t>0$, the Poisson kernel $\psi_t(x,y)$ is given by

$$\psi_t(x,y) = \frac{t}{2\pi(x^2+y^2+t^2)^{3/2}}, \quad \forall (x,y) \in \mathbf{R}^2. \tag{2.2}$$