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Colocated Finite Volume Schemes for Fluid Flows

S. Faure[1], J. Laminie[1,2] and R. Temam[1,3,*]

[1] *Laboratoire de Mathématiques, Université Paris-Sud, Bâtiment 425, 91405 Orsay, France.*
[2] *GRIMAAG Guadeloupe, Université des Antilles et de la Guyane, Campus de Fouillole, BP 592, 97157 Pointe à Pitre Cedex, France.*
[3] *Institute for Scientific Computing and Applied Mathematics, Indiana University, Bloomington, IN 47405, USA.*

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Abstract. Our aim in this article is to improve the understanding of the colocated finite volume schemes for the incompressible Navier-Stokes equations. When all the variables are colocated, that means here when the velocities and the pressure are computed at the same place (at the centers of the control volumes), these unknowns must be properly coupled. Consequently, the choice of the time discretization and the method used to interpolate the fluxes at the edges of the control volumes are essentials. In the first and second parts of this article, two different time discretization schemes are considered with a colocated space discretization and we explain how the unknowns can be correctly coupled. Numerical simulations are presented in the last part of the article. This paper is not a comparison between staggered grid schemes and colocated schemes (for this, see, e.g., [15,22]). We plan, in the future, to use a colocated space discretization and the multilevel method of [4] initially applied to the two dimensional Burgers problem, in order to solve the incompressible Navier-Stokes equations. One advantage of colocated schemes is that all variables share the same location, hence, the possibility to use hierarchical space discretizations more easily when multilevel methods are used. For this reason, we think that it is important to study this family of schemes.

AMS subject classifications: 76M12, 76D05, 68U120, 74S10, 74H15
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1 Introduction

We consider the Navier-Stokes equations in their velocity-pressure formulation and the continuity equation written for an incompressible viscous fluid; Ω is an open bounded

*Corresponding author. *Email addresses:* `sylvain.faure@math.u-psud.fr` (S. Faure), `jacques.laminie@univ-ag.fr` (J. Laminie), `temam@indiana.edu` (R. Temam)

domain in $\mathbb{R}^2$, for our simulations we use a rectangular domain $\Omega=(0,L_1)\times(0,L_2)$.
For given volume forces $\mathbf{f}=(f_u,f_v)$, we look for the velocity vector $\mathbf{u}$ and the pressure p such that:

$$\frac{\partial \mathbf{u}}{\partial t}-\nu\triangle\mathbf{u}+(\mathbf{u}\cdot\nabla)\mathbf{u}+\nabla p=\mathbf{f} \quad \text{in } \Omega\times[0,T], \tag{1.1}$$

$$div\,\mathbf{u}=0, \tag{1.2}$$

where $\nu>0$ is the kinematic viscosity and, in space dimension two, $\mathbf{u}=\big(u(x,y,t),v(x,y,t)\big)$, $t\geq 0$.

On the boundary $\partial\Omega$ of Ω, we impose a Dirichlet no-slip boundary condition:

$$\mathbf{u}\Big|_{\partial\Omega}=\mathbf{g}, \tag{1.3}$$

where $\mathbf{g}=(g_u,g_v)$ is a given function defined on $\partial\Omega$.

Traditionnally, the staggered variable arrangement was prefered to a colocated variable arrangement. Indeed, the colocated arrangements have long been considered as impracticable since these colocated schemes were known to generate a decoupling between the velocities and the pressure. This difficulty was subsequently resolved using appropriate interpolations of the fluxes (see below, e.g., (2.15)). Based on this idea, the first successful colocated finite volume schemes were introduced in 1981 by Hsu [9], Prakash [16] and Rhie [18]. A further advantage of colocated schemes is that they can be easily used for complex geometries [25]. Moreover, multilevel techniques, which result in a significant reduction of computing time on fine grids, are also much easier to apply to the colocated arrangement; this is an essential point regarding our objective to implement multilevel methods for the Navier-Stokes equations [4]. See [15, 22] for a detailed comparison between staggered and colocated schemes. We intend in a subsequent work [5] to combine the multilevel method presented in [4] with the colocated schemes described here. For theoretical aspects of the finite volume methods, see [2].

The purpose of the present article is to describe two colocated schemes associated with different time discretizations. For each scheme, we will study if there exists a decoupling between the velocities and the pressure. Then, we will comment on the numerical results obtained in the case of a driven cavity. Note that the emphasis here is on the development of the method only and therefore the driven cavity flow is not studied with too challenging values of the Reynolds number.

In the following, the domain Ω is discretized by rectangular finite volumes of same dimensions $\Delta x\Delta y$ with $M\Delta x=L_1$ and $N\Delta y=L_2$ (M, N are given integers). Hence, we have MN volumes which are defined (see Fig. 1) by:

$$\Big(K_{ij}=[x_{i-\frac{1}{2}},x_{i+\frac{1}{2}}]\times[y_{j-\frac{1}{2}},y_{j+\frac{1}{2}}]\Big)_{i=1,\cdots,M,\,j=1,\cdots,N},$$

where

$$x_{i+\frac{1}{2}}=i\Delta x \text{ for } i=0,\cdots,M,$$
$$y_{j+\frac{1}{2}}=j\Delta y \text{ for } j=0,\cdots,N.$$