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Lattice Boltzmann Schemes with Relative Velocities

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Abstract. In this contribution, a new class of lattice Boltzmann schemes is introduced and studied. These schemes are presented in a framework that generalizes the multiple relaxation times method of d'Humières. They extend also the Geier's cascaded method. The relaxation phase takes place in a moving frame involving a set of moments depending on a given relative velocity field. We establish with the Taylor expansion method that the equivalent partial differential equations are identical to the ones obtained with the multiple relaxation times method up to the second order accuracy. The method is then performed to derive the equivalent equations up to third order accuracy.

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1 Introduction

The lattice Boltzmann method (LBM) is a numerical method able to simulate many various hydrodynamic systems: fluids flows [3, 5, 15], acoustics [17], heat equation [12], multiphase fluids [4, 21], Schrödinger equation [22], for instance. It was introduced to overcome some drawbacks of the "lattice gas automata" [9, 20]. It originates from a discretization of the Boltzmann equation [6] and comes in two successive phases: an exact transport step and a relaxation step. The latter is determined by the choice of a simplified Boltzmann collision kernel. The Bhatnagar Gross Krook approach (BGK) [2] and the

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multiple relaxation times approach (MRT) [15] are two usual choices that lead to a diagonal relaxation phase with a fixed set of moments. Despite the variety of applications and domains involving the lattice Boltzmann method, some theoretical aspects remain debatable as the stability at low viscosities and the lack of Galilean invariance.

In 2006, Geier proposed the "cascaded Boltzmann automata" [10, 11]. According to [10], the lack of Galilean invariance is identified as a source of instability due to the creation of a negative numerical viscosity. Note that the equilibrium distributions are directly derived from the Maxwellian distributions, not exactly identical to the equilibrium presented in Qian et al. [6]. Some works deal with the cascaded approach: it is written as a MRT with a generalized equilibrium in [1,18]; forcing terms are included in the cascaded framework and a Chapman-Enskog expansion is presented in [18] to recover the Navier-Stokes equations.

The purpose of this paper is to generalize the d'Humières method and also the cascaded method by introducing moments that depend on an arbitrary velocity field. In the following, this approach refers to scheme with relative velocities. The collision step happens in a frame moving at a velocity $\widetilde{u}$ and not in a fixed frame. The velocity field is here considered in a general way: it is an arbitrary function of space and time. Taking $\widetilde{u}=0$ reduces the scheme to a classical d'Humières scheme and taking $\widetilde{u}$ as the fluid velocity yields to the cascaded Geier's scheme. This method provides a "cascaded triangular" structure during the transition between the fixed moments of the d'Humières method and the moments depending on the given velocity field.

Formal expansions are usually performed to characterize the limit problem simulated by the lattice Boltzmann schemes with the acoustic scaling [7], or the diffusive scaling [13]. The Taylor expansion method has been used with the acoustic scaling to derive third order equivalent equations for the general d'Humières schemes [8]. We extend this work to the schemes with relative velocities in the general non linear case. The dependence on space and time of the new velocity field is the main difficulty to circumvent.

In the first part of this paper we recall the lattice Boltzmann framework and the associated notations. We generalize it and we introduce the schemes with relative velocities. The cascaded scheme then appears as a particular scheme with relative velocities. In the second part, we study the third order consistency with the Taylor expansion method: we establish that the resulting hydrodynamic is identical to the one of Lallemand and Luo [15] up to the second order accuracy. At the third order, the additional terms depend explicitly on the given velocity field.

2 Description of the scheme

2.1 The usual framework

We consider $\mathcal{L}$, a regular lattice in d dimensions with a typical mesh size Δx. The time step Δt is determined by the acoustic scaling after the specification of the velocity scale λ