

# Simulation of Acoustic Behavior of Bubbly Liquids with Hybrid Lattice Boltzmann and Homogeneous Equilibrium Models

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**Abstract.** Homogeneous equilibrium model (HEM) has been widely used in cavitating flow simulations. The major feature of this model is that a single equation of state (EOS) is proposed to describe the thermal behavior of bubbly liquid, where both kinematic and thermal equilibrium is assumed between two phases. In this paper, the HEM was coupled with multi-relaxation-time lattice Boltzmann model (MRT-LBM) and the acoustic behavior was simulated. Two approaches were applied alternatively: adjusting speed of sound (Buick, *J. Phys. A*, 2006, 39:13807-13815) and setting real gas EOS. Both approaches result in high accuracy in acoustic speed predictions for different void (gas) volume of fractions. It is demonstrated that LBM could be successfully applied as a Navier-Stokes equation solver for industrial applications. However, further dissipation and dispersion analysis shows that Shan-Chen type approaches of LBM are deficient, especially in large wave-number region.

**AMS subject classifications:** 76Q05, 76B10, 76T10, 35Q20

**Key words:** Homogeneous equilibrium model, lattice Boltzmann method, acoustics, bubbly liquids.

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## 1 Introduction

Natural cavitation is defined as the phenomenon of the formation of vapor bubbles in a flow due to the pressure falls below the liquids vapor pressure, which can cause the falling of fluid machinery performance [1,2], or drag reducing for high speed underwater vehicles [3]. Cavitating flows in most industrial applications are turbulent and the dynamics of the interface formed involves complex interactions between the vapour and liquid phases. These interactions are not well understood in the closure region of cavities, where a distinct interface may not exist and where the flow is unsteady.

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Numerical simulation is conducted widely as a powerful tool for the cavitation study [4–6]. Among them, the one-fluid model is widely applied to the cavitation in nozzles, which is based on associated precondition method and studies of mathematical and thermodynamic properties [7–9]. In this model, the phases are assumed to be in kinematic and thermodynamic equilibrium: they share the same pressure, temperature and velocity in the sub-cell scale, namely homogeneous equilibrium model. Delannoy and Kueny [7] proposed a barotropic equation of state (EOS) in which density was a function of pressure. And therefore, the well developed compressible solver (for N-S or Euler equations) could be utilized for this hydrodynamic multiphase problem. Goncalves [9] further constructed the EOS formula to avoid infinite value of speed of sound in pure phases.

In recent decades, lattice Boltzmann methods (LBM) emerged as an attractive CFD method, which bases on the mesoscale particle dynamics [10,11]. Some sophisticated flow phenomena, such as interfacial flow, reactive flow, are simulated successfully by combined with certain particle properties, whose motion is simply divided into “collision” and “stream” loops. Considering the scientific and practical meanings, the LBM simulations to real-gas systems attract numerous attentions, which is further applied to phase transition simulations including cavitating [12] and boiling [13] phenomenon. The main advantage of LB methods for real-gas roots in its particle foundations without introducing empirical models: Chen’s results showed that LBM gave precise predictions for cavitating bubble growth by purely considering the particle kinetics issues [14]. Of course, it should be mentioned that, in real cavitating flows, the situation is much more complex than the pure phase transition due to the deficiency, like tiny bubbles, impurities, in the substances.

In this paper, we coupled homogeneous equilibrium model with lattice Boltzmann method. Through the approaches of real-gas simulations, proposed first by Shan & Chen [11], we demonstrate that LBM could be regarded as an acceptable NS equation solver. And both integral and differential equation of state result in precise results in the frame of LBM. As well, the dissipation and dispersion relations are analyzed for both ideal and non-ideal cases. The deficiency of the Shan-Chen type approaches (especially in the circumstances with large wave number) is showed according to the comparison of our results as well.

## 2 Models and numerical methods

### 2.1 Local homogeneous equilibrium model

The local homogeneous equilibrium model (HEM) roots in such assumption that thermodynamic effects are negligible, which is a common one because the heat transfer from a bubble to the surrounding liquid takes place on length scales that are on the order of micrometers and is very rapid. It is a one-fluid model, where the two fluids system is treated as one. The substance is considered as mixture of liquid and numerous tiny bubbles with certain vapor volume of fraction ( $\alpha$ ). Vaporization and condensation processes