Commun. Comput. Phys.
doi: 10.4208/cicp.081214.240515s

Vol. **18**, No. 4, pp. 931-956
October 2015

Simulation of Incompressible Free Surface Flow Using the Volume Preserving Level Set Method

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Received 8 December 2014; Accepted (in revised version) 24 May 2015

Abstract. This study aims to develop a numerical scheme in collocated Cartesian grids to solve the level set equation together with the incompressible two-phase flow equations. A seventh-order accurate upwinding combined compact difference (UCCD7) scheme has been developed for the approximation of the first-order spatial derivative terms shown in the level set equation. Developed scheme has a higher accuracy with a three-point grid stencil to minimize phase error. To preserve the mass of each phase all the time, the temporal derivative term in the level set equation is approximated by the sixth-order accurate symplectic Runge-Kutta (SRK6) scheme. All the simulated results for the dam-break, Rayleigh-Taylor instability, bubble rising, two-bubble merging, and milkcrown problems in two and three dimensions agree well with the available numerical or experimental results.

AMS subject classifications: 35Q30, 76D05, 76D27, 76M20

Key words: Level set equation, upwinding combined compact difference scheme, three-point grid stencil, minimize phase error, symplectic Runge-Kutta.

1 Introduction

It is computationally difficult to predict flow equations subject to a sharply varying interface between the air and water. Interface tracking methods encounter deformed meshes that conform to the interface in the Lagrangian sense. The meshes should, as a result, explicitly adapt to the interface. In interface capturing methods, interface is an implicit

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function defined in a fixed mesh. The advantages and disadvantages of applying the Lagrangian type of interface tracking and the Eulerian type of interface capturing methods have been reported in the literature. However, it is still difficult to conclude which methodology is superior to the other.

ALE (Arbitrary Lagrangian Eulerian) [1] method has been well known to be very efficient in modeling a small interface deformation. When interface undergoes a large deformation, a computationally very expensive re-meshing procedure is needed. Boundary integral methods [2–4] were developed to discretize equations along only the interface that separates different liquids, making this class of methods a very attractive one. However, every time when an interface has been merged or split, a time consuming effort is inevitable to continue the computation using the boundary integral method.

The Eulerian based methods in [5–12] are also applicable to the problems having not only a complicated interface but also to the problems involving physical complexities. The VOF method normally involves using a color function and has the ability of conserving the volume of each fluid phase more exactly. The other successful approach developed to model two-phase flows is known as the level set method [13, 14]. The level set function described by Sussman, Smereka and Osher [15] can be easily transported and accurately calculated, respectively. Choice of a proper signed distance function for the sake of re-shaping level set function and implementing re-initialization procedure for the purpose of enhancing numerical stability are normally required while applying level set methods [16].

Level set method applied to simulate interface is known to have the problem regarding the numerical dissipation and mass conservation. Many attempts have been made to circumvent these difficulties [17]. Among them, one can straightforwardly improve the accuracy of the level set solution by the high-order discontinuous Galerkin method [16]. For improving the mass conservation property of the level set method, one can also apply hybrid methods. This class of methods, such as the coupled level set volume of fluid (CLSVOF) method [18] and the particle level set (PLS) method [17], combines an accurate Lagrangian tracking method with the level set method [19]. It is also numerically possible to optimize the level set method by the spatially adaptive method [19] to improve mass conservation. Different adaptive mesh refinements have been conducted to improve the predicted level set solution near the interface considerably, thereby improving the degree of mass conservation. The other method known as the conservative level set method [20] solves the level set equation in conservative form. This method introduces Heaviside function to get a sharp interface approximation. Smearing of the solution with time can be avoided by applying nonlinear re-initialization equation in the simulation of level set equation. For an overview of the level set methods, one can refer to [21–25].

In this paper, a seventh-order upwinding combined compact difference scheme (UCCD7) with the smallest numerical phase error is developed for reducing the dispersion error generated from the discrepancy between the effective and actual scaled wavenumbers. This scheme can predict interface excellently and avoid mass accumulation or depletion. This paper is organized as follows. Section 2 describes the method for