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Construction, Analysis and Application of Coupled Compact Difference Scheme in Computational Acoustics and Fluid Flow Problems

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Abstract. In the present work, a new type of coupled compact difference scheme has been proposed for the solution of computational acoustics and flow problems. The proposed scheme evaluates the first, the second and the fourth derivative terms simultaneously. Derived compact difference scheme has a significant spectral resolution and a physical dispersion relation preserving (DRP) ability over a considerable wavenumber range when a fourth order four stage Runge-Kutta scheme is used for the time integration. Central stencil has been used for the present numerical scheme to evaluate spatial derivative terms. Derived scheme has the capability of adding numerical diffusion adaptively to attenuate spurious high wavenumber oscillations responsible for numerical instabilities. The DRP nature of the proposed scheme across a wider wavenumber range provides accurate results for the model wave equations as well as computational acoustic problems. In addition to the attractive feature of adaptive diffusion, present scheme also helps to control spurious reflections from the domain boundaries and is projected as an alternative to the perfectly matched layer (PML) technique.

AMS subject classifications: 65N06, 65N35, 76D05, 76D17

Key words: Coupled compact difference scheme, DRP property, adaptive numerical diffusion, computational acoustic, PML technique.

1 Introduction

Computational acoustics is an important and active research area [1–5]. Acoustic signals propagate in the form of longitudinal waves in air. Pressure fluctuations associated

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with acoustic signals are usually very small as compared to the large background pressure field. Effects of viscosity of the medium on the propagation of acoustic signal are negligibly small when the signal propagates over a short distance [6]. Calculations of acoustic problems have to be performed for a long duration avoiding numerical instabilities to obtain spectral contents of the signals [1]. In this context, the chosen discretization schemes should neither numerically amplify nor attenuate the acoustic signal. Propagation of a computed acoustic signal strongly depends on the phase, dissipation and the dispersion properties associated with individual wavenumber component for the used discretization schemes [2, 5, 7]. Hence, the accurate computation of the acoustic field is a challenging topic.

Computational aeroacoustics (CAA) problems are focused on obtaining numerical solution of formation and propagation of acoustic disturbances due to fluctuations in the fluid flow [2, 8, 9]. For an accurate numerical solution, it is necessary that the numerical scheme must resolve all the scales present in the acoustic field as well as flow field. Hence the used numerical scheme must have a higher spectral resolution. In addition, numerical propagation speed of all the resolved scales must be identical to the corresponding physical propagation speed of the individual scale in order to avoid dispersion error [10–14]. It is very much important to ensure that the used numerical scheme is capable of correctly estimating amplitudes and speed of all the fluctuations in the fluid flow which act as an acoustic source. The scheme must exhibit a better DRP nature across a large wavenumber range [12, 14–16] as compared to the traditional discretization schemes.

Compact schemes are widely used for solving computational aeroacoustics problems as well as fluid flow problems [2, 3, 17, 18]. These schemes provide excellent spectral resolution as compared to the traditional explicit discretization methods while evaluation of various derivative terms [5, 17–23]. These schemes have a relatively compact stencil as compared to the traditional explicit schemes of same order and same spectral resolution [14]. In compact schemes, derivative of a function is obtained by either solving a tri-diagonal or a penta-diagonal matrix equation [17–19]. Although compact schemes have fewer grid points in the stencil, their implicit nature accounts for the contribution from large number of nodes present in the domain which provides a higher spectral resolution even on a coarser mesh [5, 14, 17–19, 24].

Researchers have also focused their attention on the development of new DRP schemes [14,21,25]. While designing a numerical scheme, one aims for preserving physical dispersion relation across a complete wavenumber range. However, various existing numerical schemes are capable of preserving physical dispersion relation only for a limited part of the complete wavenumber spectrum. The DRP nature of the various difference schemes is usually observed for the low wavenumber components while spurious dispersion is associated with high wavenumber components [13,14]. Spurious waves which are associated with the extreme form of a dispersion error in which computed scales not only travel with a wrong velocity but also in a wrong direction are identified as the q-waves [11, 13]. These spurious waves are often responsible for numerical instabilities [13,18,26]. Numerical instabilities can be avoided by either using numerical filters [18,20,26–29] or by using