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Splitting Finite Difference Methods on Staggered Grids for the Three-Dimensional Time-Dependent Maxwell Equations

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Abstract. In this paper, we study splitting numerical methods for the three-dimensional Maxwell equations in the time domain. We propose a new kind of splitting finite-difference time-domain schemes on a staggered grid, which consists of only two stages for each time step. It is proved by the energy method that the splitting scheme is unconditionally stable and convergent for problems with perfectly conducting boundary conditions. Both numerical dispersion analysis and numerical experiments are also presented to illustrate the efficiency of the proposed schemes.

AMS subject classifications: 65N10, 65N15

Key words: Splitting scheme, alternating direction implicit method, finite-difference time-domain method, stability, convergence, Maxwell's equations, perfectly conducting boundary.

1 Introduction

In this paper we consider splitting finite difference methods for the three-dimensional Maxwell equations

$$\frac{\partial E_x}{\partial t}=\frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y}-\frac{\partial H_y}{\partial z}-\sigma E_x\right), \tag{1.1}$$

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$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \sigma E_y\right), \tag{1.2}$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma E_z\right), \tag{1.3}$$

$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} - \sigma^* H_x\right), \tag{1.4}$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} - \sigma^* H_y\right), \tag{1.5}$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} - \sigma^* H_z\right) \tag{1.6}$$

in a lossy medium with electric permittivity ε, magnetic permeability μ, electric conductivity σ and the equivalent magnetic loss rate σ^*, where $\mathbf{E} = (E_x, E_y, E_z)$ and $\mathbf{H} = (H_x, H_y, H_z)$ denote the electric and magnetic fields. If these fields (multiplied with ε and μ respectively) start out divergence free, they will remain so during wave propagation. Physically this is a consequence of the relations $div(\varepsilon\mathbf{E}) = \rho$ (where ρ is the local charge density), and $div(\mu\mathbf{H}) = 0$. Their invariance in time is also a consequence of the Maxwell equations (1.1)-(1.6) and need therefore not be imposed separately.

The numerical approximation of Maxwell's equations has emerged recently as a crucial enabling technology for radio-frequency, microwave, integrated optical circuits, antennas, and wireless engineering [1–3, 13–15, 19, 24]. The finite-difference time-domain (FDTD) method, first introduced by Yee [26] (also called Yee's scheme) and extensively utilized and refined by Taflove and others [24], has been the most widely used numerical algorithm in computational electromagnetics in the time domain over the past few decades, due to its simplicity, robustness, and low cost per grid point [24]. Yee's scheme employs a fully staggered space-time grid and is explicit with a second-order convergence rate in both time and space. The stability and convergence analysis were carried out for Yee's scheme in [20, 22] using the energy method.

However, Yee's scheme is only conditionally stable so that the time step and the spatial step sizes $\Delta t, \Delta x, \Delta y$ and Δz must satisfy the Courant-Friedrichs-Lewy (CFL) stability condition

$$\Delta t \leq \frac{1}{c}\left[\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}\right]^{-1/2}$$

in the three-dimensional case, where $c = 1/\sqrt{\epsilon\mu}$ is the wave velocity. If the time step is not within the bound, the FDTD scheme will become numerically unstable. Thus, the computation of the three-dimensional Maxwell equations by Yee's scheme will become extremely difficult when the spatial discretization step sizes become very small. To overcome this difficulty, an unconditionally stable alternating direction implicit (ADI) FDTD scheme was first proposed in [27] and [21] for the three-dimensional Maxwell equations with an isotropic medium (see also [24]). This ADI-FDTD scheme consists of only two