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A Comparative Study of Rosenbrock-Type and Implicit Runge-Kutta Time Integration for Discontinuous Galerkin Method for Unsteady 3D Compressible Navier-Stokes equations

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Abstract. A comparative study of two classes of third-order implicit time integration schemes is presented for a third-order hierarchical WENO reconstructed discontinuous Galerkin (rDG) method to solve the 3D unsteady compressible Navier-Stokes equations: — 1) the explicit first stage, single diagonally implicit Runge-Kutta (ESDIRK3) scheme, and 2) the Rosenbrock-Wanner (ROW) schemes based on the differential algebraic equations (DAEs) of Index-2. Compared with the ESDIRK3 scheme, a remarkable feature of the ROW schemes is that, they only require one approximate Jacobian matrix calculation every time step, thus considerably reducing the overall computational cost. A variety of test cases, ranging from inviscid flows to DNS of turbulent flows, are presented to assess the performance of these schemes. Numerical experiments demonstrate that the third-order ROW scheme for the DAEs of index-2 can not only achieve the designed formal order of temporal convergence accuracy in a benchmark test, but also require significantly less computing time than its ESDIRK3 counterpart to converge to the same level of discretization errors in all of the flow simulations in this study, indicating that the ROW methods provide an attractive alternative for the higher-order time-accurate integration of the unsteady compressible Navier-Stokes equations.

AMS subject classifications: 65M60, 76M10

Key words: Implicit time integration, Rosenbrock-Wanner, discontinuous Galerkin, WENO, Navier-Stokes.

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1 Introduction

The discontinuous Galerkin (DG) methods, originally introduced for solving the neutron transport by Reed and Hill [1], have become popular for the solution of systems of conservation laws in recent decades. Nowadays, they are widely used in computational fluid dynamics (CFD), computational acoustics, and computational magneto-hydrodynamics. A comprehensive overview of the DG methods can be found in [2]. The DG methods combine the attractive features of both the finite element (FE) and finite volume (FV) methods, and thus are especially advantageous in solving the hyperbolic-type system of equations in terms of solution accuracy [3–6], treatment of non-conforming meshes [7], and implementation of the hp-adaptivity [8]. However, the DG methods also have a number of weaknesses that have yet to be addressed, before they can be used for problems of practical interest. In particular, how to reduce the high computational costs and how to develop more efficient time integration methods are two of the most long-standing research challenges.

In order to reduce the high costs associated to the DG methods, Dumbser et al. [9–11] introduced a new family of so-called reconstructed DG, termed PnPm schemes and referred to as rDG (PnPm) in this paper. Pn indicates that a piecewise polynomial of degree of n is used to represent the underlying DG solution, and Pm represents a polynomial solution of degree of m ($m \geq n$) that is reconstructed from the underlying Pn polynomial and used to compute the fluxes. The PnPm schemes can be constructed based on a few different algorithms, e.g., the recovery approach [12], the reconstruction approach [13,14], and the Gauss-Green approach [15,16], all of which were proved to deliver the designed grid convergence of $\mathcal{O}(h^{m+1})$ [17]. Indeed, implicit methods can especially benefit from the use of rDG (PnPm) methods as the costs can be substantially reduced in two aspects [18, 19]. Firstly, fewer spatial integration points are required for evaluating the residual vector and Jacobian matrix. For instance, the third-order rDG (P1P2) only needs 4 points for triangular boundary integral whereas the equivalent DG (P2) requires 7. Secondly, the Jacobian matrix of rDG (PnPm) is based on the underlying DG (Pn), and thus requires much less storage than the equivalent DG (Pm). For example, the memory needed for the diagonal part of the Jacobian matrix of rDG (P1P2) is 400 word versus 2500 needed by DG (P2) for the 3D Navier-Stokes equations.

The spatial discretization of the compressible Navier-Stokes equations leads to a system of Ordinary Differential Equations (ODEs). Significant progress has also been made in developing efficient higher-order implicit time integration methods for such system in order to reduce the temporal discretization error incurred from the use of lower-order time integration methods. Bijl et al. [20] introduced ESDIRK schemes for the finite volume solutions to the Navier-Stokes equations. Later on, Wang and Mavriplis [21] extended the ESDIRK schemes to solve the compressible Euler equations using a high-order p-multigrid DG method. Xia et al. [22] also used ESDIRK for the time accurate solutions of the 3D compressible Navier-Stokes equations in the context of the reconstructed