Commun. Comput. Phys.
doi: 10.4208/cicp.080615.010216a

Vol. **20**, No. 4, pp. 870-901
October 2016

Application of the LS-STAG Immersed Boundary/ Cut-Cell Method to Viscoelastic Flow Computations

Olivier Botella*, Yoann Cheny, Farhad Nikfarjam and Marcela Stoica

LEMTA, Université de Lorraine, CNRS, 2 avenue de la Forêt de Haye – TSA 60604, 54518 Vandoeuvre Cedex, France.

Received 8 June 2015; Accepted (in revised version) 1 February 2016

Abstract. This paper presents the extension of a well-established Immersed Boundary (IB)/cut-cell method, the LS-STAG method (Y. Cheny & O. Botella, *J. Comput. Phys.* Vol. 229, 1043-1076, 2010), to viscoelastic flow computations in complex geometries. We recall that for Newtonian flows, the LS-STAG method is based on the finite-volume method on staggered grids, where the IB boundary is represented by its level-set function. The discretization in the cut-cells is achieved by requiring that global conservation properties equations be satisfied at the discrete level, resulting in a stable and accurate method and, thanks to the level-set representation of the IB boundary, at low computational costs.

In the present work, we consider a general viscoelastic tensorial equation whose particular cases recover well-known constitutive laws such as the Oldroyd-B,White-Metzner and Giesekus models. Based on the LS-STAG discretization of the Newtonian stresses in the cut-cells, we have achieved a compatible velocity-pressure-stress discretization that prevents spurious oscillations of the stress tensor. Applications to popular benchmarks for viscoelastic fluids are presented: the four-to-one abrupt planar contraction flows with sharp and rounded re-entrant corners, for which experimental and numerical results are available. The results show that the LS-STAG method demonstrates an accuracy and robustness comparable to body-fitted methods.

AMS subject classifications: 68U20, 76A10, 76D05, 76M12, 76M25

Key words: Immersed boundary method, cut-cell method, incompressible flows, viscoelastic fluids, planar contraction flows.

1 Introduction

This paper presents an immersed boundary (IB)/cut-cell finite-volume (FV) method for the computation of viscoelastic flows in 2D irregular geometries. Lately, FV methods

*Corresponding author. *Email addresses:* olivier.botella@univ-lorraine.fr (O. Botella), yoann.cheny@univ-lorraine.fr (Y. Cheny)

have drawn a renewed interest for viscoelastic computations, motivated by their inherent low computational costs compared to finite-element (FE) formulations. For Newtonian flows in irregular geometries, IB methods have now reached a high level of maturity (see [1] for a recent review). They rely on cost-effective Cartesian grid methods that alleviate the generation of body fitted meshes and the need of frequent remeshing in the case of problems with moving boundaries. IB methods have found numerous applications in turbulent flow computation [2, 3], fluid-structure interaction [4, 5], biofluid dynamics [6, 7], *etc*.. But, to our knowledge, IB methods have seldomly been applied to viscoelastic flows [8, 9].

One of the reasons might be related to the treatment of cells near the immersed boundary for highly elastic flows, which need accurate discretization due to the thin shear stress layers that develop near solid walls and geometrical singularities of the domain, and which are the main cause of numerical instabilities of viscoelastic computations [10–12]. As a matter of fact, classical IB methods such as the *momentum forcing method* [1] do not discretize flow equations in the cut-cells, *i.e.* cells that are cut by the immersed boundary, but use *ad hoc* extrapolations instead. The lack of consistency of IB discretization manifests in diverse forms, such as numerical stability issues, non divergence-free velocities, nonphysical oscillations of the pressure, or concentration of the numerical errors in the vicinity of the immersed boundary [13–15]. To overcome these shortcomings, we have proposed in [16, 17] a new IB method, called the LS-STAG method, where a unified discretization of the flow equations in both Cartesian and cut-cells is achieved by requiring that the global conservation properties of the flow (total mass, momentum and kinetic energy) be satisfied at the discrete level, resulting in a stable and accurate method and, thanks to the level-set representation of the IB boundary, at low computational costs. During the construction of this method, it was observed that the discretization of the Newtonian [17] and generalized Newtonian [18] stresses in the cut-cells was the most intricate part of the method, and prone to large errors if the discretization was not performed adequately. In this paper, we have the intention to extend these ideas on Newtonian stress discretization and global conservation properties of IB methods to the discretization of the flow equations for viscoelastic liquids.

For modelling the rheology of viscoelastic fluids, the various constitutive equations which have been devised (Upper-Convected Maxwell, Oldroyd-B, White-Metzner, Giesekus *etc*., see [12, 19] for a review) have common features: a set of nonlinear hyperbolic transport equations for the elastic part of the stress tensor, that have to be coupled to the incompressible Navier-Stokes equations. The numerical solution of such coupled problems with FE [20], FV [21, 22] or spectral [23] methods shares the same numerical difficulties, and the most well known is directly associated to the coupling of the discrete velocity and stress variables. As it is observed for the velocity-pressure coupling in incompressible Newtonian flows, the discretization of the velocity and stress has to be compatible for preventing spurious oscillations of the stress variables when the level of elasticity becomes non-negligible. For FE methods, it amounts to enforcing a