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Two-Size Moment Multi-Fluid Model: A Robust and High-Fidelity Description of Polydisperse Moderately Dense Evaporating Sprays

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Abstract. High fidelity modeling and simulation of moderately dense sprays at relatively low cost is still a major challenge for many applications. For that purpose, we introduce a new multi-fluid model based on a two-size moment formalism in sections, which are size intervals of discretization. It is derived from a Boltzmann type equation taking into account drag, evaporation and coalescence, which are representative of the complex terms that arise in multi-physics environments. The closure of the model comes from a reconstruction of the distribution. A piecewise affine reconstruction in size is thoroughly analyzed in terms of stability and accuracy, a key point for a high-fidelity and reliable description of the spray. Robust and accurate numerical methods are then developed, ensuring the realizability of the moments. The model and method are proven to describe the spray with a high accuracy in size and size-conditioned variables, resorting to a lower number of sections compared to one size moment methods. Moreover, robustness is ensured with efficient and tractable algorithms despite the numerous couplings and various algebra thanks to a tailored overall strategy. This strategy is successfully tested on a difficult 2D unsteady case, which proves the efficiency of the modeling and numerical choices.

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1 Introduction

Two-phase flows constituted of a gaseous phase carrying a disperse condensed phase play a key role in many industrial and scientific applications *e.g.* spray combustion in Diesel engines or aeronautical combustion chambers, heterogeneous energetic materials in solid rocket motors, fluidized beds, *etc*. In all these applications the disperse phase is composed of particles/droplets of various sizes that can possibly evaporate, break-up, coalesce or aggregate, and have their own inertia and size-conditioned dynamics and heating. The needs of research and industrial applications and the availability of powerful computational means appeal for high-fidelity, robust and efficient descriptions of the disperse phase in such flows.

We choose to describe the dynamics of the disperse phase in a statistical sense using a kinetic approach because of its accuracy and flexibility. The disperse phase information is completely contained in the so-called Number Density Function (NDF). The NDF measures an ensemble average (over a given set of initial conditions) number of particles at a specific location in the phase space at a given time. The phase space is determined by the number of internal coordinates that describe the particle state: position, velocity, size, temperature, *etc*. These variables evolve due to physical phenomena: transport, drag force, evaporation, heating, collisions *etc*. which are accounted for through the Williams equation [39]. Moreover, in many applications, the coordinate of the phase space that is the most essential to deal with is size, because the other internal coordinates, such as the velocity or the temperature, are strongly conditioned on it.

There are several strategies to solve this kinetic equation. Lagrangian-Monte-Carlo approach [2, 11, 17] allows to approximate the NDF by a sample of discrete numerical parcels describing particles of various internal coordinates. It is called Direct Simulation Monte-Carlo method (DSMC) in [3] and is generally considered to be the most accurate method for solving this type of equation; it is specially suited for Direct Numerical Simulations (DNS) on canonical configurations since it does not introduce any numerical diffusion. However, the number of parcels required to achieve a satisfactory statistical convergence comes to be high in 3D cases, especially for unsteady configurations, when the size distribution has to be well approximated in addition to the spatial repartition of the spray. To overcome this limitation, Eulerian methods offer a promising alternative. The main objective is then to describe both the size distribution and the velocity (and eventually the temperature) conditioned on size. The size distribution, for the spray as well as for non-inertial particles (aerosol, soots) can be modeled thanks to three types of methods: 1) the sectional methods [15, 35] introducing a discretization of the size variable into intervals called sections, 2) the moment methods (see for example [19, 26, 28, 40, 41]), which consist in writing equations on some moments of the NDF, 3) the class methods, sampling the size variable (see citations of [8] and [33]). The velocity and temperature can be easily considered by class methods. However, these methods suffer from their inability to tackle integral terms except at the cost of strong modal simplifications. For the other methods, moments are used to capture the velocity (or temperature) conditioned on size. For the