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Estimation of Impacts of Removing Arbitrarily Constrained Domain Details to the Analysis of Incompressible Fluid Flows

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Abstract. Removing geometric details from the computational domain can significantly reduce the complexity of downstream task of meshing and simulation computation, and increase their stability. Proper estimation of the sensitivity analysis error induced by removing such domain details, called *defeaturing errors*, can ensure that the sensitivity analysis fidelity can still be met after simplification. In this paper, estimation of impacts of removing arbitrarily constrained domain details to the analysis of incompressible fluid flows is studied with applications to fast analysis of incompressible fluid flows in complex environments. The derived error estimator is applicable to geometric details constrained by either Dirichlet or Neumann boundary conditions, and has no special requirements on the outer boundary conditions. Extensive numerical examples were presented to demonstrate the effectiveness and efficiency of the proposed error estimator.

AMS subject classifications: 35Q30, 49M05, 76D05

Key words: Defeaturing error, adjoint formulation, incompressible fluid mechanics.

1 Introduction

Performing sensitivity analysis to predict the physical performance or behavior of a design component in science and engineering is a very time-consuming task in spite of the increasing computational power. The difficulties of this challenge lie in the increasing geometry complexity of modern engineering designs and the original non-linearity of the underlying physical phenomenon. For example, a typical automobile consists of

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about 3000 parts and the Boeing 777 over 100,000 [4]. On the other hand, various physical phenomena essentially contain nonlinear principles, ranging from gravitation to fluid dynamics, which are much more difficult to compute than the linear ones. Generally, each of these nonlinear phenomena has to be studied as a separate problem.

Traditional approaches to accelerating the speed of sensitivity analysis in engineering practice can be classified into two main categories: to reduce the geometric complexity of the underlying computational domains and to use simpler underlying PDEs. The latter has been widely investigated recently, and mainly involves a process of a posteriori estimation of *modeling errors*, caused by the use of unperfect or simplified mathematical equations to represent physical phenomena [25]. Based on such error estimations, strategies of adaptive modeling are then applied to generate simpler modeling equations, until a certain sensitivity analysis accuracy can be finally achieved. The adaptive modeling process is similar to that of the well studied topic of adaptive meshing based on the result of a posterior estimation of the *numerical approximation errors*, incurred in computing discrete solutions for boundary value problems [12,15,18,19,26,29] using finite element methods (FE), finite volume methods or mesh-free methods and so on.

The approaches to a posteriori estimation of the modeling errors or the numerical approximation errors generally assume that the underlying geometry in which analysis is performed remains unchanged (up to some tolerance) before and after approximation. However, in practical sensitivity analysis fields, a process of geometry simplification, or defeaturing, is usually conducted before meshing to convert the fully detailed design to a simplified geometry. The aim is to remove design features, such as holes, slots or blends that have little impact on the results of analysis, allowing the analysis to be performed more quickly on a simpler model. Such irrelevant geometric details can significantly increase the time and computational complexity both for the meshing process and the analysis computation performed on it [28,36]. Worse, they may even lead to mesh generation failure [36] or ill-conditioned computations [28] that may produce inaccurate analysis results. In extreme cases, the fully featured problem may be too complex for sensitivity analysis to be tractable, and model simplification is essential in such cases.

The benefits of suppressing features however come at a cost of difference between the solutions on the simplified geometry and those on the original geometry, called *defeaturing error*. Understanding such defeaturing error is essential to ensure that the desired analysis accuracy can still be met after defeaturing. Previous approaches on numerical error estimations or modeling error estimations, which assume that the underlying geometry upon which analysis is performed is exact, are thus cannot be directly employed here.

Estimations of the novel defeaturing error are rarely addressed, except [17], until recently. Suresh and his colleagues [34,35] first developed a series of approaches, called *feature sensitivity analysis* there, to handle the issue of defeaturing error estimations. Later on, Ferrandes et al. [10] and Li et al. [20] developed different approaches to further address this important issue. However, these previous research efforts are mainly limited to the cases of linear problems, e.g., Poisson problem, linear elasticity or plate bending, and