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Simulation of Maxwell's Equations on GPU Using a High-Order Error-Minimized Scheme

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Abstract. In this study an explicit Finite Difference Method (FDM) based scheme is developed to solve the Maxwell's equations in time domain for a lossless medium. This manuscript focuses on two unique aspects – the three dimensional time-accurate discretization of the hyperbolic system of Maxwell equations in three-point non-staggered grid stencil and it's application to parallel computing through the use of Graphics Processing Units (GPU). The proposed temporal scheme is symplectic, thus permitting conservation of all Hamiltonians in the Maxwell equation. Moreover, to enable accurate predictions over large time frames, a phase velocity preserving scheme is developed for treatment of the spatial derivative terms. As a result, the chosen time increment and grid spacing can be optimally coupled. An additional theoretical investigation into this pairing is also shown. Finally, the application of the proposed scheme to parallel computing using one Nvidia K20 Tesla GPU card is demonstrated. For the benchmarks performed, the parallel speedup when compared to a single core of an Intel i7-4820K CPU is approximately 190x.

AMS subject classifications: 35Q61, 65M06, 65Y05

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1 Introduction

Gauss's law is known to serve as the constraint equation on the Maxwell's equations consisting of the Faraday's and Ampère's laws. The two divergence-free equations in Gauss's law are not always discretely satisfied when solving the electromagnetic (EM) wave solutions solely from the Faraday's and Ampère's equations. The computed nonzero-divergence errors in magnetic and electric flux densities can introduce undesired instability into the calculation process. Circumvention of this type of unphysical oscillations is therefore crucial in a successful simulation of Maxwell's equation [1]. The two constraint equations in Gauss's law can be numerically satisfied at all times when solving Maxwell's equations using Yee's staggered grid system [2]. For efficiently calculating EM wave solutions in parallel, in this study an explicit scheme capable of rendering a set of divergence-free electric and magnetic solutions is adopted using non-staggered (or co-located) grids.

While approximating the spatial and temporal derivative terms in Maxwell's equations, the key measure of the prediction accuracy is the dispersion error introduced into the solution [3]. In the worst cases, dispersion error tends to hinder simulation of problems involving narrow pulses and large time spans [4]. Therefore, any attempts to numerically solve the Maxwell equations should endeavor to simultaneously reduce dispersion and dissipation errors – this is particularly important for approximation of the first-order spatial derivative terms in the EM wave equations. Owing to this reason, extensive effort has been put toward the development of higher order FDTD schemes. One can refer to the detailed overview of higher order time-domain methods in [5]. An alternative to reduce numerical dispersion error, which constitutes a major source of error in the FDTD method, is to design schemes based on optimization criteria, other than to maximize the order of accuracy. Minimization of dispersion error can be achieved through the angle-optimized FDTD algorithm [6] and the parameter-dependent FDTD scheme [3]. Over the past two decades, there have been several FDTD schemes developed with the objective of satisfying the dispersion-relation equation to reduce dispersion errors [7]. Shlager and Schneider compared dispersion properties of several low-dispersion FDTD algorithms [8].

When attempting long-term simulations – meaning simulations which require a very large number of discrete time steps – of the electromagnetic wave equations, the solution quality may be deteriorated substantially due to the accumulation of numerical error resulting from the application of a non-symplectic time-stepping scheme. These accumulated errors, while initially quite small, may build to values large enough to physically alter the properties of the solution. In order to avoid this problem, it is important to numerically preserve the symplectic property existing in Maxwell's equations when treating the time derivative terms in Faraday's and Ampère's equations. When simulating Maxwell's equations, the quality of the solution predicted by the Finite Difference Time Domain (FDTD) method can be deteriorated as well by the introduced anisotropy error. Dispersion and anisotropy errors are both accumulative with time and can seriously con-