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Second-Order Two-Scale Computational Method for Nonlinear Dynamic Thermo-Mechanical Problems of Composites with Cylindrical Periodicity

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Abstract. In this paper, a novel second-order two-scale (SOTS) computational method is developed for nonlinear dynamic thermo-mechanical problems of composites with cylindrical periodicity. The non-linearities of these multi-scale problems were caused by the temperature-dependent properties of the composites. Firstly, the formal SOTS solutions for these problems are constructed by the multiscale asymptotic analysis. Then we theoretically explain the importance of the SOTS solutions by the error analysis in the pointwise sense. In addition, a SOTS numerical algorithm is proposed in detail to effectively solve these problems. Finally, some numerical examples verify the feasibility and effectiveness of the SOTS numerical algorithm we proposed.

AMS subject classifications: 65M60, 35Q74, 41A60, 74Q10

Key words: Second-order two-scale, nonlinear dynamic thermo-mechanical problems, temperature-dependent properties, cylindrical periodicity.

1 Introduction

In recent years, composite materials have been widely used in engineering applications owing to their excellent physical properties compared to the traditional single component materials, especially in aerospace field. In engineering applications these composites are usually served under complex thermo-mechanical environments. Because of a great application prospect, the thermo-mechanical performances of composite materials have been a research hotspot of scientists and engineers. And it is important to compute the thermo-mechanical responses of the composites in engineering applications.

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It is known to all that the materials have the temperature-dependent properties when serving in high-temperature circumstances (see [1–3, 26–28, 30]). At this situation, the temperature-dependent properties of the composites should be taken into account in order to perform a more accurate analysis. Up to now, some studies have been performed on dynamic thermo-mechanical problems of composites. Most of these studies focused on one-way thermo-mechanical coupling problems (see [1–4,6–10]), namely only the thermal field affects the mechanical field. Besides, some researchers devoted to the two-way thermo-mechanical coupling problems which are fully coupled hyperbolic and parabolic systems, but their researches were based on the cartesian coordinate system and also didn't consider the temperature-dependent nonlinear effect of material properties [11–15]. In the last few years, some research results for composites with cylindrical geometry structures have appeared [7–10, 24, 32]. To the best of our knowledge, there is a lack of adequate researches on nonlinear dynamic two-way thermo-mechanical problems of composites with cylindrical periodicity (see Fig. 1).

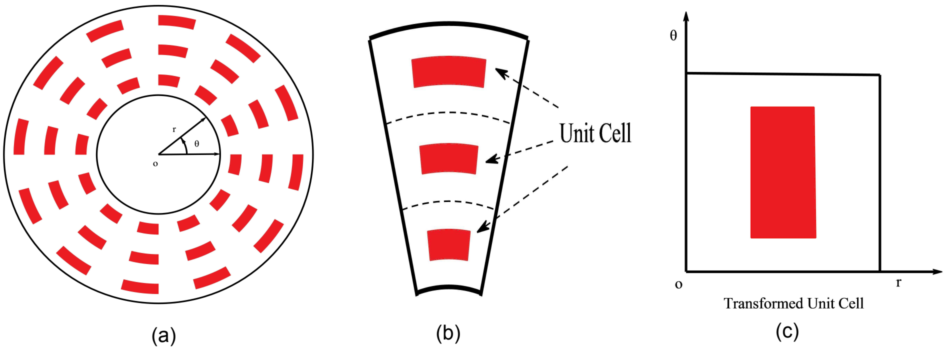

Figure 1: (a) Actual physical area; (b) Actual unit cell; (c) Transformed unit cell Y.

In the past few decades, mathematicians and engineers have developed some multiscale methods to study the multiscale behaviors of the composite materials, such as the asymptotic homogenization method (AHM), heterogeneous multiscale method (HMM), variational multiscale method (VMS), and multiscale finite element method (MsFEM) [29], etc. The AHM is a kind of mathematical method which has a rigorous mathematical foundation and can combine with finite element method very well. So it is widely used to analyze the physical and mechanical behaviors of the composites. We refer the interested readers to [4, 8–10, 29]. However, they only considered the first-order asymptotic expansions. In recent years, Cui et al. introduced the second-order two-scale (SOTS) analysis method [5, 12–14, 16, 17, 33, 34] to accurately predict the physical and mechanical behaviors of composites. By the second-order correctors, the micro-scale fluctuation information of the composites can be captured more precisely.

The aim of this paper is to develop a SOTS computational method for nonlinear dynamic thermo-mechanical problems of composites with cylindrical periodicity. The nonlinearities of these multiscale problems were caused by the temperature-dependent prop-