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Domain Decomposition Preconditioners for the System Generated by Discontinuous Galerkin Discretization of 2D-3T Heat Conduction Equations

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Abstract. In this paper we are concerned with numerical methods for nonlinear time-dependent problem coupled by electron, ion and photon temperatures in two dimensions, which is called the 2D-3T heat conduction equations. We propose discontinuous Galerkin (DG) methods for the discretization of the equations. For solving the resulting discrete system, we employ two domain decomposition (DD) preconditioners, one of which is associated with the non-overlapping DDM and the other is based on DDM with small overlap. The preconditioners are constructed by dropping the couplings between particles and each preconditioner consists of three preconditioners with smaller matrix size. To gauge the efficiency of the preconditioners, we test two examples and make different settings of parameters. Numerical results show that the proposed preconditioners are very effective to the 2D-3T problem.

AMS subject classifications: 35K51, 35K55, 65M55, 65M60

Key words: 2D-3T heat conduction equations, discontinuous Galerkin method, domain decomposition preconditioner, iterative numbers.

1 Introduction

Unsteady radiation flow equations arising from the classical conservational theorem of mass, momentum and energy are the basic control partial differential equations for laser driven implosion of a fuel capsule with the goal of igniting a self-sustained reaction in inertial confinement fusion experiments. In most cases, we can simplify them into 2D-3T

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heat conduction equations to approximately describe the radiation evolution of energy across multiple materials and perceive the exchange of energy among electrons, ions and photons (cf. [14]). There are many methods, such as finite difference method and finite volume method, to discretize the 2D-3T equations (cf. [1–3, 21, 22]). Because of strong nonlinearity and multi-scale properties of the equations, the preconditioning of the resulting systems was particularly studied. In [14] and [23], a parallel adaptive multigrid algorithm for the 2D-3T heat conduction equations was proposed. In [20], an algebraic two-level iterative method was discussed. More methods such as multigrid Newton-Krylov method [15] and preconditioned Jacobian-free Newton-Krylov method [10] were also introduced. In this paper, we propose two preconditioners based on the domain decomposition method.

Discontinuous Galerkin methods have been widely used to the discretization of elliptic and parabolic problems (cf. [6, 16, 18] and references therein). It is known that discontinuous Galerkin methods have obvious advantages over other types of finite element methods. For example, the trial and test spaces are very easy to construct; they can naturally handle inhomogeneous boundary conditions and curved boundaries; they perform very well in dealing with equations with discontinuous coefficients. For these reasons, we propose discontinuous Galerkin method for the discretization of 2D-3T heat conduction equations with multi-medium and strong discontinuous coefficients.

Since the purpose of this paper is to construct preconditioners for 2D-3T problems, we will try to develop efficient preconditioners for the system generated by DG discretizations of the heat conduction equations. In [4, 5, 8, 9] and [12], non-overlapping and overlapping domain decomposition preconditioners for the systems generated by DG discretizations of some other equations have been studied. In [11], a non-overlapping domain decomposition preconditioner with simple coarse solver is proposed for solving elliptic equation discretized by conforming finite elements. Detailed introductions for domain decomposition methods can be found in [19]. In this paper, both non-overlapping and overlapping domain decomposition preconditioners will be constructed. In order to reduce the cost of calculation, we do not directly construct domain decomposition preconditioners for the original (coupled) system, instead, we firstly eliminate the coupling of the temperatures among electron, ion and photon and then construct three independent domain decomposition preconditioners for the diagonal blocks, which correspond to the temperatures of electron, ion and photon, respectively. Due to the special features of the discontinuous Galerkin method, it is quite easy and flexible to construct valid space decompositions for the methods, especially for the non-overlapping Schwarz methods. For both preconditioners, the same simple coarse solver associated with the coarse triangulation is proposed, and slightly different local solvers defined on subdomains are introduced. We apply the GMRES method and the preconditioned GMRES method with the proposed preconditioners to solve the discrete system generated by DG discretizations of 2D-3T heat conduction equations. To gauge the efficiency of the proposed preconditioners, we test examples with one-dimensional and two-dimensional boundary conditions, and implement the preconditioned GMRES methods with various settings in the precon-