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An Implicit Unified Gas Kinetic Scheme for Radiative Transfer with Equilibrium and Non-Equilibrium Diffusive Limits

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Abstract. This paper is about the construction of a unified gas-kinetic scheme (UGKS) for a coupled system of radiative transport and material heat conduction with different diffusive limits. Different from the previous approach, instead of including absorption/emission only, the current method takes both scattering and absorption/emission mechanism into account in the radiative transport process. As a result, two asymptotic limiting solutions will appear in the diffusive regime. In the strong absorption/emission case, an equilibrium diffusion limit is obtained, where the system is mainly driven by a nonlinear diffusion equation for the equilibrium radiation and material temperature. However, in the strong scattering case, a non-equilibrium limit can be obtained, where coupled nonlinear diffusion system with different radiation and material temperature is obtained. In addition to including the scattering term in the transport equation, an implicit UGKS (IUGKS) will be developed in this paper as well. In the IUGKS, the numerical flux for the radiation intensity is constructed implicitly. Therefore, the conventional CFL constraint for the time step is released. With the use of a large time step for the radiative transport, it becomes possible to couple the IUGKS with the gas dynamic equations to develop an efficient numerical method for radiative hydrodynamics. The IUGKS is a valid method for all radiative transfer regimes. A few numerical examples will be presented to validate the current implicit method for both optical thin to optical thick cases.

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Key words: Radiative transfer, asymptotic preserving, implicit method, multiscale modeling.

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1 Introduction

This paper is about the construction of unified gas kinetic scheme (UGKS) for a system with coupled radiative transfer and material temperature evolution equations. For the radiative transport part, besides capturing absorption/emission mechanism in the previous UGKS method, the scattering process is taken into account as well in this paper. As a coupled system, for the low opacity material, such as the case with small absorption/emission coefficient and small scattering coefficient, the interaction between the radiation and material is weak, and the radiation propagates in a transparent way with particle-type behavior, the so-called optical thin regime. In this regime, the numerical method for radiation is basically to solve the streaming transport equation with upwind approach, such as ray tracking in SN methods. For a high opacity material, which has large absorption/emission coefficient or large scattering coefficient, the severe energy exchange between the radiation and material heat evolution makes photon's mean free path diminish. As a result, different asymptotic limits in the optical thick regime will appear. In the case with large absorption/emission coefficients, an equilibrium diffusive process for radiation will emerge and the material temperature and the radiation temperature will get the same value. The previous UGKS solves the equations with the absorption/emission only and captures the above diffusive limit [14, 15]. However, for the case with highly scattering material, the diffusive limiting equation will still include different temperatures for the radiation and material, with a coupled temperature evolution equation in the so-called non-equilibrium diffusive regime [19, 20]. In this paper, the previous UGKS will be extended to capture such a non-equilibrium diffusive regime. More importantly, besides recovering different limiting regimes, the aim of UGKS is to present accurate solutions in all regimes efficiently from the transparent to the diffusive evolutions. Different regimes in UGKS are identified automatically through the the ratio of the local cell size to the local photon's mean free path, with a smooth transition across all transport regimes. The UGKS framework was developed in [12]. The previous UGKS studies for radiative transfer include the linear radiation transport model [10], or the gray and frequency-dependent radiative transfer systems [14, 15]. The critical step in UGKS is to construct a time evolving solution for the radiation intensity at a cell interface for the flux evaluation. Depending on the ratio of the time cell size to the photon's mean free path, the evolution solution covers all regimes from the kinetic scale free transport to the hydrodynamic diffusive evolution. As a result, the cell size and time step in UGKS are not limited by the photon's mean free path and collision time [1, 3]. The efficiency of the UGKS in different regimes is achieved by solving the coupled macroscopic evolution equations for the radiation energy and material thermal energy first, before updating the source terms in the discretized radiative transport equation.

In the previous UGKS for radiative transfer, the numerical time step is controlled by the CFL condition, which is determined by the ratio of the cell size over the speed of light. In order to couple the radiative transfer equation with hydrodynamic equations, this small time step restriction for radiation will constrain severely the computational ef-