

# A Moving-Least-Square Immersed Boundary Method for Rigid and Deformable Boundaries in Viscous Flow

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**Abstract.** We present a moving-least-square immersed boundary method for solving viscous incompressible flow involving deformable and rigid boundaries on a uniform Cartesian grid. For rigid boundaries, noslip conditions at the rigid interfaces are enforced using the immersed-boundary direct-forcing method. We propose a reconstruction approach that utilizes moving least squares (MLS) method to reconstruct the velocity at the forcing points in the vicinity of the rigid boundaries. For deformable boundaries, MLS method is employed to construct the interpolation and distribution operators for the immersed boundary points in the vicinity of the rigid boundaries instead of using discrete delta functions. The MLS approach allows us to avoid distributing the Lagrangian forces into the solid domains as well as to avoid using the velocity of points inside the solid domains to compute the velocity of the deformable boundaries. The present numerical technique has been validated by several examples including a Poiseuille flow in a tube, deformations of elastic capsules in shear flow and dynamics of red-blood cell in microfluidic devices.

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**Key words:** Moving least squares, immersed boundary method, direct-forcing method, Navier-Stokes equations, cell sorting device.

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## 1 Introduction

Viscous incompressible flows involving deformable boundaries and complex geometries are encountered in many engineering applications. In recent years, many Cartesian-grid methods have been developed for solving these types of problems to avoid the need of

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a re-meshing process. Among these developments, the Immersed Boundary (IB) method has shown great promises in handling flows with moving boundaries. The IB method was originally proposed by Peskin to study the blood dynamics in human heart [1, 2]. The IB method was further developed for many flow problems involving deformable boundaries [3–6] as well as rigid boundaries [7–9]. Lai and Peskin [7] used a stiff-spring forcing formulation to enforce the noslip condition at the solid boundaries. The stiff-spring approach requires a small time-step to preserve the stability. To avoid using small time-step, implicit forcing schemes [9, 10] were proposed to enforce the noslip boundary condition. An alternative approach to improve the time-step is the direct forcing formulation approach employed in [11–17] where the velocity boundary condition is imposed through an interpolation procedure. In the direct forcing method, forcing terms are enforced at the grid points next to the rigid boundaries. In other similar direct forcing approaches [8, 18], the forcing terms are applied directly at the positions of the rigid boundaries and are computed based on the momentum equations and the desired velocity at the rigid boundaries.

Other Cartesian-grid methods for complex geometries include cut-cell finite-volume approach [19–21], ghost-cell approach [22] and curvilinear IB method [23–25]. In the cut-cell approach, grid cells are re-shaped according to the local geometry if they are cut by the rigid boundaries. The gradients and fluxes across the faces of these cut cells are approximated using polynomial interpolating function to preserve second-order accuracy. Most applications of the cut-cell approach focused on 2D problems because the complex cut-cell procedure was not straightforward in three dimensions. In the ghost-cell method, ghost cells are defined as solid cells that have neighbor cells in the fluid domain and their ghost-values are extrapolated to enforce implicitly the noslip condition at the boundary. In the curvilinear IB approach [23–25], boundary conditions are reconstructed at grid nodes of a background curvilinear grid generated to closely approximate the shape of an arbitrarily complex solid boundary. More details of these methods can be found in [26, 27].

In the present work, we propose an IB method for solving the incompressible Navier-Stokes equations for viscous flow involving both rigid and deformable boundaries. Our approach for handling rigid boundaries is largely based on the direct forcing method [11–17] in which noslip boundary condition is taken into account implicitly by reconstructing the velocities at the fluid nodes in the immediate neighbor of the solid boundaries. In the previous direct-forcing method [11–17], the reconstructed velocity is calculated by linear interpolation along specific directions such as grid lines or the normal directions. Here, we propose a reconstruction approach that utilizes Moving Least Squares (MLS) method [28] to reconstruct the velocity near the solid boundaries. MLS method allows us to include information from all spatial directions and from more points in both fluid and rigid boundaries in the reconstruction procedure. The MLS reconstruction has been used in the context of embedded-boundary formulations to handle rigid boundaries [29, 30]. In the present study, we also use MLS interpolation procedure together with the IB method to handle deformable boundaries in the presence of solid boundaries. In the