

A Second Order Ghost Fluid Method for an Interface Problem of the Poisson Equation

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Abstract. A second order Ghost Fluid method is proposed for the treatment of interface problems of elliptic equations with discontinuous coefficients. By appropriate use of auxiliary virtual points, physical jump conditions are enforced at the interface. The signed distance function is used for the implicit description of irregular domain. With the additional unknowns, high order approximation considering the discontinuity can be built. To avoid the ill-conditioned matrix, the interpolation stencils are selected adaptively to balance the accuracy and the numerical stability. Additional equations containing the jump restrictions are assembled with the original discretized algebraic equations to form a new sparse linear system. Several Krylov iterative solvers are tested for the newly derived linear system. The results of a series of 1-D, 2-D tests show that the proposed method possesses second order accuracy in L^∞ norm. Besides, the method can be extended to the 3-D problems straightforwardly. Numerical results reveal the present method is highly efficient and robust in dealing with the interface problems of elliptic equations.

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Key words: Interface problem, ghost fluid method, Poisson equation, jump conditions.

1 Introduction

The elliptic equation with a discontinuous physical field across the irregular interface appears in many applications such as diffusion phenomenon, heat transfer, crystal growth and many others. For fluid dynamic problems, the method used for treating irregular interface can be extended in solving the incompressible NavierStokes equations. For instance, without adding source terms, the effects of surface tension can be considered in the pressure Poisson equation straightforwardly.

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To solve an elliptic equation with irregular interfaces, a body-fitted grid can be applied [1]. Unfortunately, generating mesh that fits the boundaries of the computational domain with complex internal geometries is time-consuming, often requiring manual intervention to modify and cleaning-up the geometry. On the other hand, we can use the finite element method [2] with more flexible unstructured mesh to model the complex boundary. However, it may require a huge amount of time for the regeneration or deformation of the computational grid when the corresponding interfaces are changing. The Cartesian grid method can be generated automatically and efficient in handling the complex geometries with simplified data structure and formulations. One difficulty in using the Cartesian grid method exists in how to impose the jump conditions implicitly at the grid points adjacent to the interface without losing accuracy.

The IB (immersed boundary) method is a type of Cartesian grid method first proposed by Peskin [3] for the simulation of human heart. The direct forcing approach was latter proposed by Mohd-Yusof [4]. However, high order immersed boundary methods are restricted to certain type of boundary conditions. To ensure numerical stability, the traditional IB method uses Heaviside function to smooth the jumps of the diffusive coefficient which may bring out unexpected smearing around the interface. With the application of generalized Taylor expansions, the original IIM (Immersed Interface Method) [5,6] adaptively modifies the stencil to obtain the $\mathcal{O}(h)$ truncation error along the interface. For smooth coefficients, this reduces to the standard 5-point finite difference stencil. IIM is a second order numerical method to preserve the jump conditions at the interface. Compared with the original second-order IIM [5], a newly developed IIM [7] achieves arbitrarily high-order accuracy with a wider set of grid stencils. However, this algorithm is fairly complex and result in a non-symmetric and not diagonally dominated system.

The Ghost Fluid Method (GFM) was proposed by R. Fedkiw et al. [8] to properly treat the boundary conditions across the interface. The GFM creates an artificial fluid to implicitly enforce proper conditions. Motivated by the original GFM, Liu et al. [9] introduce fictitious points along coordinates to enforce the jump conditions properly. Although a symmetric positive definite linear system is derived, the tangential flux $[\beta \nabla u \cdot \tau]$ in determining the fictitious contribution is neglected, which results in only first order accuracy. The MIB (Matched Interface and Boundary) method [10] was then proposed to account for a non-zero $[\beta \nabla u \cdot \tau]$ by differentiating the given jump conditions using one-sided interpolations. This treatment widens the stencil in several directions that depend on the local geometry, and results in a non-symmetric discretization. The MIB method was extended by Zhou et al. [11] to handle high curvature geometry, and by Yu et al. [12] to provide a 3D version MIB method.

Some researchers use the fictitious points to enforce the Dirichlet or Neumann type boundary conditions as well as the jump conditions for the immersed interface boundary. The method presented by Johansen et al. [13] achieves second order accuracy and preserves jumps at the interface. However, this method only handling the Dirichlet type boundary conditions. In the method proposed by Cisternino et al. [14], additional unknowns are introduced to allow straightforward expression of the interface transmission