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WENO Scheme-Based Lattice Boltzmann Flux Solver for Simulation of Compressible Flows

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Abstract. In this paper, the finite difference weighed essentially non-oscillatory (WENO) scheme is incorporated into the recently developed lattice Boltzmann flux solver (LBFS) to simulate compressible flows. The resultant WENO-LBFS scheme combines advantages of WENO scheme and LBFS, e.g., high-order accuracy, high resolution and physical robustness. Various numerical tests are carried out to compare the performance of WENO-LBFS and that of other WENO-based flux solvers, including the Lax-Friedrichs flux, the modification of the Harten-Lax-van Leer flux, the multi-stage predictor-corrector flux and the flux limiter centered flux. It turns out that WENO-LBFS is able to capture discontinuous profile in shock waves. Comparatively, the WENO-LBFS scheme costs less CPU time and eases programming.

AMS subject classifications: 76N99

Key words: WENO scheme, lattice Boltzmann flux solver, numerical flux, shock wave, compressible flows.

1 Introduction

Computational fluid dynamics (CFD) is the product of the modern fluid mechanics, numerical fluid and computer science. It takes the computer as the tool and applies various discrete formats, numerical experiments, computer simulations and analytical studies to solve practical problems. At present, CFD technology is widely used in the civil engineering fields, long-span Bridges, the design of the aircraft and jet engines, water conservancy engineering, etc. The basic equations of computational fluid dynamics can be divided into compressible flow and incompressible flow and can also be divided into the

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inviscid flow (Euler equations) and viscous flow (Navier-Stokes equations). The compressible flow is widely applied in combustion science, space flight, jet engines, heat transfer, gas turbine, etc. Usually, the shock wave may exist in compressible problems. Therefore, how to develop high-order accuracy and high-resolution numerical schemes is a valuable and challenging research work.

In the past few decades, numerical algorithms for compressible flow have been continuously developed. After von Newmann [17] proposed numerical dissipation format to solve strong intermittent flow problems, Godunov [35] first proposed discontinuous decomposition algorithm in 1959. Godunov algorithm can effectively reflect the physical characteristics of flow, but it only has first order accuracy, ineffective calculation of contact discontinuity, complex programming and intensive computation [11]. Later, van Leer extended Godunov scheme to obtain MUSCL scheme [22]. MUSCL scheme has second order accuracy, but it can not avoid numerical oscillations. Subsequently, Colella and Woodward [32] developed third order Godunov scheme that is called PPM. PPM has a significant improvement in accuracy and resolution, and it can better capture shock waves. In 1973, Discontinuous Galerkin (DG) method was introduced by Reed and Hill [39]. DG method is a kind of non-standard finite element method (FEM), which combines the advantages of finite difference method (FDM) and finite volume method (FVM). DG method can conveniently achieve the adaptive computing, and it can also construct the format of the arbitrary order accuracy [30]. Unfortunately, all these high-order schemes have to introduce a slope limiter to avoid numerical oscillations. In the late 1980s, an essentially non-oscillation (ENO) scheme was proposed by Harten [1]. ENO scheme eases restrictions of non-growing total variation and allows to increase small total variation. What's more, it can achieve consistently high order accuracy, and it can also avoid numerical oscillations. However, it should be mentioned that only the smoothest stencil is adopted to construct ENO scheme and other stencils are abandoned. Basing on that, Liu and Osher et al. [40] proposed a weighted ENO (WENO) scheme using a convex combination of all candidate stencils. WENO scheme has significantly improved in accuracy and resolution comparing with ENO scheme. In the past few decades, finite difference WENO (FD-WENO) scheme [7, 10, 15] and finite volume WENO (FV-WENO) scheme [4, 7, 16, 31] have greatly developed. But Shu [7] strongly suggests using FD-WENO scheme in practice owing that it has high order accuracy, high resolution and a few calculations.

It is generally known that the local Riemann solver is the key of efficient numerical schemes to solve compressible flow problems. First, we briefly review some conventional fluxes to approximate the solution of Riemann problem. The Godunov flux presented by Godunov [35], which is based on the exact Riemann solver, has the smallest numerical viscosity among all monotone fluxes. But the Godunov flux often does not exist explicit formulas to result in a large amount of calculation. The Lax-Friedrichs (LF) flux as an efficient solver to approximate solution of Riemann problem was given by Lax and Friedrichs. It is the most widely used in many numerical schemes for its simplicity. However, the LF flux has a large numerical viscosity which results in the corresponding