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Lattice Boltzmann Method for Wave Propagation in Elastic Solids

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Abstract. We present a new lattice Boltzmann formulation for elastic solids and analyse the issue of conservation laws for solids. This new formulation allows tunable Poisson ratio. As a first benchmark problem, we consider an elastic wave propagating in an infinite homogeneous medium with point force and point dipole Ricker wavelet as the source. The method is successful in simulating behavior of elastic wave motion as a function of Poisson ratio.

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1 Introduction

Over the past few decades, substantial effort has been devoted towards improving algorithms to make realistic direct numerical simulation of complex flow physics feasible [1–6]. In this regard, the Lattice Boltzmann (LB) Method [7] has emerged as an important tool to simulate hydrodynamics as well as coupled fluid-solid problems such as suspension mechanics [2]. A common feature in most of these works is neglect of solid elasticity and associated two-way coupling between solid and fluids. However, this coupling between the fluid and wall dynamics, and the elasticity of the flexible surface results in the difference between flow through flexible tubes and rigid tubes [8]. For example, the transition between the laminar and the turbulent flow is affected by this coupling [9]. Thus, incorporation of the solid elasticity in fluid-solid coupled models is

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needed for a number of practical applications. A few important examples of fluid flow in flexible channels/tubes are: blood flows through arteries and veins [10] and flow through polymer membranes and matrices encountered in bio-technological processes [11].

Recently, a number of works have attempted to solve coupled fluid-solid [12,13] problems using finite differences for linear and non-linear elasticity. However, almost all LB related work on the moving solid investigate the problem in the limit of rigid solid. This is largely due to the fact that unlike finite differences, LB formulation for solid elasticity is not well established so far. If solid mechanics can also be solved efficiently using LBM, it would be quite convenient as the solver will be similar for both fluid and solids. However, only a few initial works have considered basic LB formulation for Poisson solids [14] i.e. limited to Poisson ratio 0.25. It can be easily said, that despite initial efforts, so far LB [15] for solid mechanics is in quite rudimentary stage.

We present a new LB formulation for elastic solid and analyse the issue of conservation laws for solids. We show that LB formulation for solid elasticity requires the extended set of conservation laws. Unlike fluid solvers, when we consider non-viscous solids, stress conservation needs to be imposed on LB formulation. We will present preliminary results with this model of elastic solid where model allows tunable Poisson ratio. The paper is organized as follows. Section 2 gives description of linear hypoelastic solids, which is followed by the section on conservation laws. Section 4 discusses the LB formulation of linear isotropic hypoelastic solids, followed by a section on LB of P and S waves. Finally, we give conclusions with a direction on future work.

2 Linear isotropic hypoelastic solids

In continuum mechanics, one usually uses one of two descriptions: the Lagrangian description and the Eulerian description for the motion [16]. While the Eulerian description is typically used for fluid dynamics, for solids traditionally one often uses Lagrangian description. However, an LB formulation of linear elasticity would require reformulation of solid mechanics in the Eulerian framework. The Eulerian approach for the solid mechanics was recently also used in the finite difference literature [17]. In Eulerian formulations, the fixed computational grid, can be exploited for the cases of extreme loading, resulting in a better stability compared to Lagrangian implementation, making them applicable to solid mechanics.

In the Eulerian framework of the solid mechanics, it is convenient to work with so called hypoelastic material for which stress does not depend on deformation tensor but the stress rate tensor is written in terms of the strain rate tensor. Before formulating LB for hypoelasticity, we briefly review hypoelasticity and its relationship with hyperelastic materials in linear regime. The basic assumptions in the hypoelastic model are [18]:

1. The stress tensor $\mathbf{P}$ depends only on the order in which the body has occupied its past configurations, but not on the time rate at which these past configurations were traversed.