Commun. Comput. Phys.
doi: 10.4208/cicp.2018.hh80.03

Vol. **24**, No. 4, pp. 1169-1195
October 2018

An Efficient Tailored Finite Point Method for Rician Denoising and Deblurring

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Received 17 June 2017; Accepted (in revised version) 12 November 2017

Abstract. In this paper, we propose using the tailored-finite-point method (TFPM) to solve the resulting parabolic or elliptic equations when minimizing the Rician denoising model developed by Getreuer et al. in [10] using augmented Lagrangian methods (ALM). Different from traditional finite difference schemes, TFPM employs the method of weighted residuals with collocation technique, which helps get more accurate approximate solutions to the equations and thus reserve more details in restored images. Numerical experiments demonstrate that with the new schemes the quality of restored images has been improved. Besides these, the existence of the minimizer of the Rician denoising model have also been established in this paper.

AMS subject classifications: 65M32, 68U10, 65K10

Key words: Rician denoising, deblurring, variational model, augmented Lagrangian methods, tailored finite point method.

1 Introduction

Magnetic resonance images (MRI) are often corrupted by Rician noise. This type of noise is signal dependent, and its distribution is different from Gaussian, Poisson or Laplace. The Fourier inverse transform involves a nonlinear operation which is applied to the original data and maps the original Gaussian distribution of the noise to a Rician distribution [23]. The Rician noise tends to be Gaussian distributed when the ratio ν/σ is high, where ν and σ are two parameters associated with the probability density function of Rician distribution (Eq. (2.2)), and it is approximately Rayleigh distributed when this ratio is low, while for those medium values of ν/σ, neither Gaussian nor Rayleigh could provide a good approximation of the distribution of the Rician noise [21].

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In the literature, there are many approaches or models that have been developed for image denoising problems. One of the most popular and powerful variational models was proposed by Rudin, Osher, and Fatemi (ROF), where the total variation was used as a regularizer. The remarkable feature of the ROF model lies in the ability of keeping object boundaries. Even since then, the total variation has been widely adapted to different imaging tasks. Other approaches include wavelet transform approaches [31], the anisotropic diffusion filter [9], Perona-Malik filter [30], and nonlinear diffusion filters [22], etc.

To remove the specific Rician noise in MRI, a few models have been proposed [10,23]. Among them, an effective total variation based variational model was proposed by Getreuer et. al. in [10]. The non-differentiability of the total variation is an issue of minimizing the ROF model efficiently. Recently, there are lots of fast algorithms introduced for this model, including the primal-dual method by Chan et al. [4], Chambolle's method [3], the split Bregman by Osher et al. [29], and the augmented Lagrangian method by Wu and Tai [37]. When the split Bregman or augmented Lagrangian method is applied, several parabolic or elliptic type equations need to be solved, and usually standard schemes are often employed. For instance, to solve linear elliptic equations, the five-point scheme is widely utilized for discretizing the Laplacian operator. This scheme, in some sense, uses polynomials to approximate the solutions of those equations, which is surely subject to the loss of accuracy. In this paper, we plan to incorporate a special numerical scheme, which is called tailored finite point method (TFPM) [15], to get more accurate solutions to the Rician denoising model [10]. This could help improve the quality of restored images.

The TFPM was originally proposed by Han, Huang, and Kellogg for a singular perturbation problem [16] in 2008. To resolve the boundary/interior layers, the traditional numerical methods need very fine mesh size to achieve a satisfactory result. These layers are characterized by rapid transitions in the solution, and are difficult to capture in a numerical approximation without using a large number of unknowns. Also, such layers tend to cause spurious oscillations in a numerical solution to the problem if the mesh size is not small enough. TFPM ensures the effectiveness in solving singularly perturbed problems [14, 19] and the non-equilibrium radiation diffusion equations [18]. The essential idea of the TFPM is that by selecting the appropriate basis functions, the solutions of the (reduced) PDE can be approximated locally. As a result, TFPM differs from the standard schemes only in the coefficients of the unknown variables at those stencils. As the new scheme builds in the properties of the solution, this modification helps maintain essential features of the original equations and also obtain more accurate approximation to the original problem. Therefore, TFPM is able to achieve higher accuracy than standard methods but at a reasonable cost.

The outline of our paper is as follows. We first review the Rician denoising model [10] and then discuss the existence of the minimizer of this model in Section 2, and recall the TFPM in Section 3. In Section 4, we provide the details of applying TFPM for solving the resulting elliptic or parabolic equations when minimizing the Rician denoising and de-