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Numerical Simulations for the Quasi-3D Fluid Streamer Propagation Model: Methods and Applications

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Abstract. In this work, we propose and compare four different strategies for simulating the fluid model of quasi-three-dimensional streamer propagation, consisting of Poisson's equation for the particle velocity and two continuity equations for particle transport in the cylindrical coordinate system with angular symmetry. Each strategy involves one method for solving Poisson's equation, a discontinuous Galerkin method for solving the continuity equations, and a total variation-diminishing Runge-Kutta method in temporal discretization. The numerical methods for Poisson's equation include discontinuous Galerkin methods, the mixed finite element method, and the least-squares finite element method. The numerical method for continuity equations is the Oden-Babuška-Baumann discontinuous Galerkin method. A slope limiter for the DG methods in the cylindrical coordinate system is proposed to conserve the physical property. Tests and comparisons show that all four strategies are compatible in the sense that solutions to particle densities converge. Finally, different types of streamer propagation phenomena were simulated using the proposed method, including double-headed streamer in nitrogen and SF_6 between parallel plates, a streamer discharge in a point-to-plane gap.

AMS subject classifications: 65Z05, 65M06, 68U20

Key words: Streamer discharge, mixed finite element method, least-squares finite element method, discontinuous Galerkin method, fluid model.

1 Introduction

A streamer is a type of electrical discharge emerging when a strong electric field is applied to a gap, e.g., in air gas. Streamers appear in nature as well as in many industrial applications, such as ozone generation, air purification, and plasma-assisted combustion.

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Because streamers develop within a very short time, e.g., nanoseconds or microseconds, it is difficult to measure all the micro-physical parameters by performing experiments. This motivated researchers to use numerical simulations to study the physics of streamer discharges [1–12].

The simplest model for simulating streamer propagation is a fluid model with the assumption of angular symmetry. This model consists of two continuity equations for particle densities coupled with Poisson's equation for the electric potential and electric field that determines the velocity of the particles [5]:

$$\begin{cases} \frac{\partial\sigma}{\partial t}+\frac{1}{r}\nabla\cdot(r\mu_\sigma\sigma\mathbf{E}-r\mathbb{D}\nabla\sigma)=S|\mathbf{E}|e^{K/|\mathbf{E}|}\sigma, & (r,z)\in\Omega,\ t>0; \\ \frac{\partial\rho}{\partial t}+\frac{1}{r}\nabla\cdot(r\mu_\rho\rho\mathbf{E})=S|\mathbf{E}|e^{K/|\mathbf{E}|}\sigma, & (r,z)\in\Omega,\ t>0; \\ -\frac{1}{r}\nabla\cdot(r\nabla\phi)=\rho-\sigma,\mathbf{E}=-\nabla\phi, & (r,z)\in\Omega,\ t>0, \end{cases} \tag{1.1}$$

where the operator $\nabla\cdot\mathbf{V}$ is defined as $(\frac{\partial V_r}{\partial r}+\frac{\partial V_z}{\partial r})$, and the computational domain is generally defined by

$$\Omega=\{(r,z):0\le r\le R,0\le z\le f(r)\}.$$

In the above so-called quasi-three-dimensional (quasi-3D) model, σ and ρ are the rescaled densities of electrons and positive ions, respectively; ϕ and $\mathbf{E}$ denote the electric potential and electric field, respectively; $\mu_\sigma=-1$, μ_ρ signify rescaled mobility constants for electrons and positive ions, respectively; D represents the rescaled diffusion coefficient for electrons. The remaining two rescaled parameters, S and K, are defined as $S=APx_0$ and $K=\frac{BPx_0}{V_0}$, respectively, where x_0 and V_0 are the length scale and applied potential scale, respectively; P is the pressure in torr, and A and B are two constants.

A Dirichlet boundary condition according to the applied voltage, is assigned to the Poisson equation, and homogeneous Neumann boundary conditions are assigned to the continuity equations.

Previous studies show that Poisson's equation occupies most of the computational time [8] and many methods have been attempted—for instance, the finite volume method used by U. Ebert, D. Bessières et al. [5,7] and the discontinuous Galerkin methods introduced by D. Arnold, M. Wheeler et al. [13–15]. Because the electric field, rather than the electric potential, couples with the continuity equations, it is natural to seek a numerical method that can directly derive a high-accuracy solution for the electric field. To do so, we refer to the mixed finite element method (MFEM) [16,17] and least-squares finite element method (LSFEM) [18,19]. Both methods rewrite Poisson's equation as a first- order system, where the electric potential and field become two independent variables. However, these methods have to solve more degrees of freedom, which requires additional computational time.

It is worth emphasising that Poisson's equation determines the velocity of the particles in the continuity equations. Thus, the compatibility of methods for Poisson's equation and the continuity equations is an important issue [20].

The above continuity equations are convection-dominated if the source terms are not taken into consideration. As is well known, traditional linear schemes for convec-