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A Hybrid Finite Element-Laplace Transform Method for the Analysis of Transient Electromagnetic Scattering by an Over-Filled Cavity in the Ground Plane

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Abstract. A hybrid finite element-Laplace transform method is implemented to analyze the time domain electromagnetic scattering induced by a 2-D overfilled cavity embedded in the infinite ground plane. The algorithm divides the whole scattering domain into two, interior and exterior, sub-domains. In the interior sub-domain which covers the cavity, the problem is solved via the finite element method. The problem is solved analytically in the exterior sub-domain which slightly overlaps the interior sub-domain and extends to the rest of the upper half plane. The use of the Laplace transform leads to an analytical link condition between the overlapping sub-domains. The analytical link guides the selection of the overlapping zone and eliminates the need to use the conventional Schwartz iteration. This dramatically improves the efficiency for solving transient scattering problems. Numerical solutions are tested favorably against analytical ones for a canonical geometry. The perfect link over the artificial boundary between the finite element approximation in the interior and analytical solution in the exterior further indicates the reliability of the method. An error analysis is also performed.

AMS subject classifications: 35M10, 65M60

Key words: Overfilled cavity, time domain electromagnetic scattering, Laplace transform, finite element method.

1 Introduction

In the last decades, the time-domain finite element method (TDFEM) has evolved into a powerful numerical tool making it possible to solve a variety of complicated electromagnetic problems (see, e.g., [1–9]). The TDFEM applied to scattering by cavities are reported

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in [10–15]. It is well known that TDFEM is relatively time consuming since it needs to solve a linear system at each time step. Others have attacked the transient scattering problems with integral equations techniques [16], and series representation approach [17]. Recently, the domain decomposition (DD) method has emerged as a promising technique supplementing the TDFEM [18–20]. The DD method typically partitions the simulated area into a number of sub-domains and Lagrange multipliers are used to enforce the interface continuity condition. Traditionally, the DD method calls for the iterative matching of the link condition (Schwartz link [21]), which incurs significant computational expense.

In [22], we solve the scattering problem in the frequency domain using a hybrid boundary integral - finite element method (BI-FEM) (see, also, [23, 24] and the references therein for this hybrid method). In this technique, an artificial boundary, typically a semi-circle, is placed over the cavity and the interior numerical solution is matched with the exterior analytical solution via the introduction of a boundary operator. The boundary operator gives rise to a non-standard boundary condition on the semi-circle which demands the implementation of a boundary array. An additional hurdle associated with the hybrid method in solving time domain problems stems from the convolution operation. The time-dependency of the artificial boundary requires the storage of historic contributions of the field.

In this paper, the DD technique is integrated into the BI-FEM. In this hybrid approach, the computational domain is partitioned into two sub-domains such that the interior problem is solved numerically and exterior analytically. Specifically, we incorporate three techniques to solve the time domain scattering problem: 1) The hybrid BI-FEM method, in which an analytical solution defined in the exterior domain is used to match the boundary condition of the interior numerical model; 2) DD method, where the whole domain is divided into two sub-domains and the link condition over the overlapped zone between the interior and exterior domains is matched; 3) The Laplace transform (LT) method, here a time dependent kernel function is obtained enabling the explicit evaluation of the boundary condition and thus avoiding the expensive Schwartz iteration. Our algorithm is implemented using the *pdetool* under the GUI environment of Matlab. We developed an analytical solution for a canonical model and compared it against our numerical results. We also plotted the linkage between the exterior analytical solution and the interior numerical one. The overall good agreement between analytical and numerical solutions gives confidence of the reliability of our fast algorithm. The accuracy of our method is further demonstrated by a mesh size related error analysis.

2 Problem setting

Mathematical treatment of time harmonic electromagnetic scattering from cavities are reported in [25] for over-filled cavities, and [26–28] for non-protruding cavities. A detailed mathematical description of the variational method for transient electromagnetic