

## Numerical Simulation of an Aortic Flow Based on a HLLC Type Incompressible Flow Solver

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Received 25 October 2007; Accepted (in revised version) 1 May 2008

Communicated by Kun Xu

Available online 15 July 2008

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**Abstract.** In this study, a three-dimensional artificial compressibility solver based on the average-state Harten-Lax-van Leer-Contact (HLLC) [13] type Riemann solution is first proposed and developed to solve the time-dependent incompressible flow equations. To implement unsteady flow calculations, a dual time stepping strategy including the LU decomposition method is used in the pseudo-time iteration and the second-order accurate backward difference is adopted to discretize the unsteady flow term. Also a third-order accurate HLLC numerical flux is derived for approximating the inviscid terms. To verify numerical accuracy, flows over a lid-driven cavity and an oscillating flat plate are chosen as the benchmark tests. In addition, the current solver is extended to solve blood flows in a realistic human aorta measured from MRI (Magnetic Resonance Imaging). The simulation geometry was derived from a three-dimensional reconstruction of a series of two-dimensional slices obtained in vivo. Numerical results demonstrate wall stresses were highly dynamic, but were generally high along the outer wall in the vicinity of the branches and low along the inner wall, particularly in the descending aorta. The maximum wall stress distribution is presented on the aortic arch in the systole. In addition, extensive counter-clockwise secondary flows and three-dimensional helical vortex influenced considerably by the presence of vessel contraction, torsion and the branches were shown in the descending aorta in the late systole and early diastolic cycles.

**AMS subject classifications:** 76D05, 92C35, 74F10

**Key words:** HLLC, Riemann solver, incompressible flow, aortic flow, fluid-structure interaction.

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## 1 Introduction

One of the popular numerical algorithms for the calculations of incompressible flows is based on Chorin's artificial compressibility method [1]. In recent years, the artificial compressibility strategy has been applied by Pan and Chakravarthy [2], Rogers and Kwak [3], Chen et al. [4], using an implicit line relaxation or LU decomposition scheme in pseudo time and applying Roe type Riemann solver based on high-order upwinding or weighted ENO interpolation on the spatial differencing. The development of the artificial compressibility method has been verified and agreed with several validated data ranging from analytical solutions of simple flows to measured data of complicated transient vortex shedding. However, the computing efficiency of three-dimensional flow problems is still an unsolved issue due to the tedious calculations on the high-order implicit and explicit numerical inviscid and viscous fluxes. To improve numerical efficiency and simplicity, Belov et al. [5] used an explicit artificial compressibility preconditioning with Jameson's artificial viscosity method to construct numerical fluxes for hyperbolic type incompressible flow equations. They utilized second-order backward difference to discretize the physical time term and a rational fourth order Runge Kutta scheme to proceed the subiterations in each pseudo-time step, thus allowing for the computations to march toward a steady state during every physical time step and achieve accurate unsteady flow calculations. In addition to Jameson type explicit preconditioning, Jin and Xu [6,7] recently provide another alternative to simulate the low Reynolds number flow based on gas-kinetic BGK model. Their gas-kinetic solver has been developed in a moving frame on the unified coordinates of Hui [8] and achieved many successful calculations of the flows with free surface and moving boundaries in the incompressible laminar flow regime. However, the applicability of gas-kinetic type solver on the calculations of complex incompressible flows such as vortex shedding or turbulence is still unknown and needs more verification. Up to the present, even so many existing hyperbolic type incompressible flow solvers have been developed, the Roe type approximate Riemann solver may be the most widely used one to solve the artificial compressibility based incompressible equations. In our previous works [9], we have developed a unified artificial compressibility solver based on the Roe type numerical flux under the hybrid Eulerian-Lagrangian coordinates to simulate the moving body flows. Our preliminary results have verified the accuracy and robustness of the unified artificial compressibility solver on many steady and transient cases. However, the Roe type solver is very time-consuming and inefficient, especially on the calculations of the three-dimensional unsteady flow problems.

In this study, we propose an alternative artificial compressibility solver based on average-state HLLC Riemann solutions originated from Toro's theoretical analysis [10]. First, the concept of average-state approximations of Riemann problems was introduced by Harten, Lax, and van Leer [11]. There is a large hierarchy of numerical flux which arises from this approach, all of which may be applied directly to the Euler equations without the need for additional ad hoc modifications. Unfortunately, most of these ap-