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Numerical Simulations of Fiber Sedimentation in Navier-Stokes Flows

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Abstract. We perform numerical simulations of the sedimentation of rigid fibers suspended in a viscous incompressible fluid at nonzero Reynolds numbers. The fiber sedimentation system is modeled as a two-dimensional immersed boundary problem, which naturally accommodates the fluid-particle interactions and which allows the simulation of a large number of suspending fibers. We study the dynamics of sedimenting fibers under a variety of conditions, including differing fiber densities, Reynolds numbers, domain boundary conditions, etc. Simulation results are compared to experimental measurements and numerical results obtained in previous studies.

AMS subject classifications: 76D05, 76M25

Key words: Fiber sedimentation, immersed boundaries, Navier-Stokes equations.

1 Introduction

The sedimentation of solid particles can be found in many natural phenomena and industrial applications, e.g., the flow of pollutants in rivers and in the atmosphere, the clarification of liquids, and the separation of particles of differing masses. Nonetheless, despite the wide applicability of particle sedimentation, some of its fundamental properties remain to be understood. One of the challenges in understanding these properties lies in the long-range hydrodynamic interactions among the particles.

Much effort has been directed to studying the sedimentation process of spherical particles. For instance, the mean sedimentation velocity and the velocity fluctuations of the particles have been investigated experimentally [27] and theoretically [3,11] (and see references therein). In contrast, the sedimentation of non-spherical particles is less well-understood. Owing to the anisotropic nature of the particles, the suspension structure

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and fiber orientation have a significant impact on the sedimentation velocity. Also, unlike spheres, isolated fibers may have motion in the direction perpendicular to gravity.

Previous fiber experiments have measured the settling speed and orientation of sedimenting marked fibers. Experiments by Herzhaft et al. [17] revealed a dilute regime of inhomogeneous sedimentation in which the fibers tend to align in the direction of gravity and to clump together in packets. Experiments by Turney et al. [33] and by Anselmet [1] suggests that, in the semi-dilute regime, fiber settling is hindered at sufficiently high volume fractions.

Although not as extensively investigated as spherical particles, there have been several numerical studies on fiber suspensions. Butler and Shaqfeh [9] and Fan et al. [13] presented numerical simulation to fiber suspensions and compared their results to experimental data. Tornberg and co-workers developed a numerical method, based on a non-local slender body approximation that yields a system of coupled integral equations, to simulate rigid and flexible fiber suspensions at zero Reynolds number regime [31,32]. Their simulation results show settling and clustering behaviors consistent with experimental measurements previously reported. In addition, Kuusela et al. [20] performed numerical calculations of the dynamics of sedimenting prolate spheroids at a low Reynolds number 0.3.

A goal of this work is to study the behaviors of sedimenting fibers under differing conditions, including differing Reynolds number flows, fiber densities, boundary conditions, etc. To achieve that goal, we formulated a mathematical model of sedimenting thin, rigid fibers as a two-dimensional (2D) immersed boundary problem. The immersed boundary approach was first developed by Peskin [26] for solving the full incompressible Navier-Stokes equations with moving boundaries. The immersed boundary method was originally developed for studying blood flow through a beating heart [25], but has since been applied to a wide variety of problems, including inner ear fluid dynamics [5], bacterial swimming [12], sperm motility in the presence of boundaries [14], cilliary beating [7], flow through an arteriole [2], etc. In particular, Stockie [28] applied this method to simulate the motion of a single pulp fiber in a linear shear flow. The immersed boundary approach offers much flexibility: the Reynolds number can be set to be any arbitrary value, the immersed boundaries as well as the fluid domain can theoretically be of arbitrary shape, and there is little constraint on the domain boundary conditions. In the current problem, the fibers are modeled as immersed boundaries that exert forces on the surrounding fluid when subjected to gravity.

Using the immersed fiber model, we study, in terms of sedimentation velocity and fiber distribution, the effects of fiber density, initial fiber distribution, Reynolds number, and domain boundary conditions. In general, our simulation results are consistent with experimental observations reported in literature.

The paper is organized as follows. In Section 2, an immersed boundary model is introduced. In Section 3, numerical simulations are performed using the method described in Section 2 for fibre suspensions in a fluid with a nonzero Reynolds number. Some discussions are presented in the final section.