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Numerical Simulation of Rarefied Gas Flows with Specified Heat Flux Boundary Conditions

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Abstract. We investigate unidirectional rarefied flows confined between two infinite parallel plates with specified heat flux boundary conditions. Both Couette and force-driven Poiseuille flows are considered. The flow behaviors are analyzed numerically by solving the Shakhov model of the Boltzmann equation. We find that a zero-heat-flux wall can significantly influence the flow behavior, including the velocity slip and temperature jump at the wall, especially for high-speed flows. The predicted bimodal-like temperature profile for force-driven flows cannot even be qualitatively captured by the Navier-Stokes-Fourier equations.

AMS subject classifications: 76P05, 82B40

Key words: Thermal boundary condition, rarefied gas flow, S model, discrete velocity method.

1 Introduction

In a broad range of engineering applications involving gas flows the characteristic spatial scale is comparable to the mean free path of the working gas. These applications range from high altitude and high speed space vehicles [1] to Micro-Electro-Mechanical Systems (MEMS) [2]. In such conditions, typically, the conventional Navier-Stokes-Fourier (NSF) equations are inappropriate as rarefaction effects become substantial. Kinetic methods such as direct simulation Monte Carlo (DSMC) [3] and direct solution of the Boltzmann equation [4] become necessary. However, the computational costs of these methods are high. In particular, as the signal/noise ratio is typically low for flows in MEMS the

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computational cost can be prohibitive for DSMC. Various techniques have been proposed to improve computational efficiency, e.g. [5–8].

Thermal management is an aspect of gas flow applications, e.g., re-entry vehicle thermal protection systems in which insulation techniques are used. To describe such systems, it is important to be able to employ constant heat flux boundary conditions so that the heat exchange between the system and the surroundings can be controlled [9–18]. If the surface is well insulated, the adiabatic boundary condition may be appropriate for flow modeling. For this purpose, it is convenient to set the accommodation coefficient in the commonly used Maxwell-type boundary condition to zero, i.e., full specular reflection; see the discussion in [11] and an example in [15]. However, to achieve full specular reflection, the surface should be perfectly smooth at the microscale. For general surfaces, the specified heat flux boundary condition should be achievable even with full diffuse-reflection. A few implementations of this have been explicitly discussed for the DSMC method [9, 10]. However, there is a lack of such discussion for simulations where the kinetic equations themselves are directly solved, even for simple flows.

In this paper, we report our numerical investigations of rarefied gas flows confined between two parallel infinite plates. We solve the Shakhov model (S model) of the Boltzmann equation [19, 20]. While one plate is specified with a constant heat flux (only zero heat flux is simulated in this paper), the other has a fixed temperature. Here we suppose that the zero heat flux boundary closely resembles the adiabatic boundary condition[§]. The numerical method we use is based on the deterministic discrete velocity method (see [21] and references therein). Both a Couette flow and a force-driven Poiseuille flow are examined.

2 Formulation of the problem

2.1 Specification

A monatomic gas is confined between two parallel infinite plates located at $y=0$ and $y=L$. The upper plate ($y=L$) has a fixed temperature, T_0, and the lower one ($y=0$) has a zero heat flux. In Couette flow, the two plates move oppositely with the same speed U_w. For Poiseuille flow, the gas is subject to a uniform external force in the x direction, i.e. the direction parallel to the plates.

2.2 Model

In order to capture rarefaction effects, we solve the S model of the Boltzmann equation, which is given as:

[§]Under rarefied conditions, it becomes rather difficult to precisely define the adiabatic boundary condition, in particular for flows with a moving wall. There may be different kinds of definitions, e.g. [22]. However, we may always use a specified heat flux boundary condition (and not necessarily set to be zero) to describe phenomenologically an "adiabatic" boundary in a practical circumstance.