Commun. Comput. Phys.
doi: 10.4208/cicp.230714.100315a

Vol. **18**, No. 5, pp. 1234-1263
November 2015

A Polynomial Chaos Method for Dispersive Electromagnetics

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Received 23 July 2014; Accepted (in revised version) 10 March 2015

Abstract. Electromagnetic wave propagation in complex dispersive media is governed by the time dependent Maxwell's equations coupled to equations that describe the evolution of the induced macroscopic polarization. We consider "polydispersive" materials represented by distributions of dielectric parameters in a polarization model. The work focuses on a novel computational framework for such problems involving Polynomial Chaos Expansions as a method to improve the modeling accuracy of the Debye model and allow for easy simulation using the Finite Difference Time Domain (FDTD) method. Stability and dispersion analyzes are performed for the approach utilizing the second order Yee scheme in two spatial dimensions.

AMS subject classifications: 35Q61, 65C99, 65M06, 65M12

Key words: Maxwell's equations, Debye polarization, relaxation time distribution, FDTD.

1 Introduction

A fundamental question in electromagnetics is how to model dispersion and dissipation of the fields in complex materials such as biological tissue. This has most often led to the use of Maxwell's equations coupled with constitutive relationships for polarization. The problem is even more difficult with noisy data or variability (heterogeneity) in the material being interrogated. Some deterministic models have been generalized to an extent that they seem to account for this variability, but there is some question as to whether the resulting models are even physically realistic.

A recently rediscovered modeling framework allows uncertainty at the molecular level through distributions of parameters representing molecular variability. Intensive experimental efforts have been pursued in describing data for complex materials in the frequency domain with distributions of dielectric parameters, especially relaxation times in multiple Debye models. A significant amount of this work is reviewed in the survey paper by Foster and Schwan [27]. The corresponding time-domain inverse problems

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were initially developed in [5] and examples for a one dimensional case were solved in [6] using finite elements for the forward simulation and quadrature for computing the expected value over a distribution. Our contribution here is to describe an approximation approach, which utilizes the generalized Polynomial Chaos framework, in two and three dimensions. This allows the efficient use of the finite difference time domain (FDTD) method for solving the dispersive Maxwell's Equations, eliminates the need for a separate computation of expected values, and simplifies analysis of the model and numerical methods. The resulting approach can be applied to realistic two and three dimensional models of human tissue or geological media in an electromagnetic interrogation inverse problem.

1.1 Background

Ultra-short pulsed electromagnetic fields are used in a wide variety of applications such as radar, environmental and medical imaging to evaluate the internal structure of objects for detecting and characterizing anomalies. This is done by studying changes in the dielectric properties (such as permittivity, conductivity, and relaxation times) of the materials under consideration for example in the noninvasive evaluation of tissues and organs in cancer detection, and nondestructive investigation of materials for cracks. Time domain numerical simulations of wave propagation in dispersive materials can help in understanding the short-pulse response of these media, and can be used to obtain threshold levels of safe exposure of humans to high energy electromagnetic fields. These simulations can also be used in inverse problem formulations for natural resource exploration, among other applications. Thus, the numerical and computational analyses of such systems can provide great benefit in real world applications.

Microwave imaging for breast cancer detection has been considered in a numerical context by several researchers (c.f., [25,55,56] and references therein) due to the enhanced contrast that can be extracted through numerical inverse problem formulations over signal processing approaches. Likewise, there is active interest in microwave and other frequency remote sensing of the environment for accurate modeling (e.g., of ocean currents), or resource exploration (c.f., [24,47] and references therein).

The detection, reconstruction, and characterization of objects in dielectrics using electromagnetic interrogation has been the subject of intense research for at least the past two decades. Numerical approaches generally fall into two categories: frequency-domain and time-domain.

We consider here broadband pulse (c.f., [70]) interrogation so as to excite multiple poles in the dielectric targets and to better distinguish between materials of interest. The use of multiple frequencies prohibits effective use of frequency domain methods, thus time domain approaches will be considered in modeling the interrogation problem. We note that frequency domain inverse problems may still be helpful in providing a baseline or initial approximation in an iterative inverse problem formulation.

In the context of time-domain forward simulations, the two most popular approaches