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Analyses and Applications of the Second-Order Cross Correlation in the Passive Imaging

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Abstract. The first-order cross correlation and corresponding applications in the passive imaging are deeply studied by Garnier and Papanicolaou in their pioneer works. In this paper, the results of the first-order cross correlation are generalized to the second-order cross correlation. The second-order cross correlation is proven to be a statistically stable quantity, with respective to the random ambient noise sources. Specially, with proper time scales, the stochastic fluctuation for the second-order cross correlation converges much faster than the first-order one. Indeed, the convergent rate is of order $\mathcal{O}(T^{-1+\alpha})$, with $0<\alpha<1$. Besides, by using the stationary phase method in both homogeneous and scattering medium, similar behaviors of the singular components for the second-order cross correlation are obtained. Finally, two imaging methods are proposed to search for a target point reflector: One method is based on the imaging function, and has a better signal-to-noise rate; Another method is based on the geometric property, and can improve the bad range resolution of the imaging results.

AMS subject classifications: 35L10, 35R30, 35R60, 78A46

Key words: Second-order cross correlation, ambient noise sources, stationary phase method, passive imaging.

1 Introduction

It is well known that the full Green's function between two passive sensors could be estimated by cross correlating wave signals emitted by ambient noise sources and recorded

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by the corresponding sensors [26, 28, 29]. The ambient noise is generated by sources that are randomly distributed in space and are statistically stationary in time. The relation between the cross correlation and Green's function can be proven using the Helmholtz-Kirchhoff identity when the noise sources surround the observation region [3, 5, 24].

In geophysics, the ambient seismic noise is used to extract the Green's function, i.e., cross-correlating the seismic noise recorded by two stations to get the Green's function between these two stations, thus to improve the knowledge about subsurface structures, see [20] and references therein. This idea has been successfully applied to background velocity estimation and travel time estimation. For background velocity estimation, the first application was carried out in seismology to obtain an estimation of the background surface wave velocity map of a large part of the Earth, where the seismic stations were used as the passive senors, and the noise sources came from the nonlinear interaction of the ocean swell with the coast that generates surface waves [25]. It has also been applied to background velocity estimation from regional to local scales [19, 23], volcano monitoring [10, 11], and petroleum prospecting [12]. For travel time estimation, it was firstly proposed in helioseismology and seismology [13, 21, 22], where the cross correlation was used to retrieve the travel time information. Besides, the cross correlation has also been used in the noise source localization [1].

Another important application of the cross correlation is passive imaging, where only passive sensors are used to search for a target reflector buried in the homogeneous or scattering medium [4]. Specially, in Garnier's and Papanicolaou's pioneer works [14–16], the first-order cross correlation and corresponding applications in the passive imaging were deeply studied, and a self-consistent theoretical framework for the analysis of first-order cross correlation based interferometric imaging techniques was introduced, which was an extension of the coherent interferometric (CINT) imaging techniques [7, 8]. And the cross correlation has also been applied to the imaging problems in the random waveguide [2].

In this paper, based on Garnier's and Papanicolaou's framework [14–16], we consider the analyses and applications of the second-order cross correlation. The idea of the second-order cross correlation firstly appeared in Garnier's and Papanicolaou's work [14]. However, few theoretical analyses have been made, and there are no applications of the second-order cross correlation in the passive imaging. The main objective of our work is to present detailed and systemic analyses on the second-order cross correlation, and to apply it to the passive imaging. The rest of the paper is organized as follows. In Section 2, the results of the first-order cross correlation are recalled. Then, based on the first-order cross correlation, the second-order cross correlation is introduced in Section 3. Specially, we prove that the second-order cross correlation is a statistically stable quantity. In Section 4 and Section 5, with the stationary phase method, we analyze the behaviors of the second-order cross correlation in both homogeneous and scattering medium. Finally, two imaging methods are proposed in Section 6. One method is based on the imaging function, which is a direct generalization of the imaging method for the first-order case. Another method is based on the geometric property, which is original in this paper.