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Chebyshev-Legendre Spectral Domain Decomposition Method for Two-Dimensional Vorticity Equations

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Abstract. We extend the Chebyshev-Legendre spectral method to multi-domain case for solving the two-dimensional vorticity equations. The schemes are formulated in Legendre-Galerkin method while the nonlinear term is collocated at Chebyshev-Gauss collocation points. We introduce proper basis functions in order that the matrix of algebraic system is sparse. The algorithm can be implemented efficiently and in parallel way. The numerical analysis results in the case of one-dimensional multi-domain are generalized to two-dimensional case. The stability and convergence of the method are proved. Numerical results are given.

AMS subject classifications: 65M12, 65M70

Key words: Multi-domain spectral method(spectral element method), Chebyshev-Legendre spectral method, vorticity equations.

1 Introduction

Spectral method has so called "infinity order" convergence rate but there are also some limits in applications. One main obstacle of spectral method is of treating discontinuous problems. The other is of treating complex domain problems. Therefore, how to overcome these obstacles has become the focus and hotspot in the research of the spectral method in recent years. Some progress has been achieved so far. For example, to the discontinuous problems, appropriate filter and reconstruction method is already used to restore the spectral approximation accuracy [1]. For the complex domain problems, if the whole domain is mapped into a regular area, it may increase the complexity of the problem and bring more errors. Then the domain decomposition skills were introduced. The basic idea of the domain decomposition method is to divide the original domain into several sub areas. If necessary, the sub-domain is mapped into regular area. And then using

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the spectral method in every sub-domain. Domain decomposition methods have been extensively used for numerical simulations involving spectral methods, as they can offer more flexibility in treating complicated domains as well as computational savings. Even for simple domain there are some good reasons to employ the domain decomposition spectral methods. Firstly, the spectral collocation points can be distributed a little more freedom so that the resolution on each subinterval can be chosen suitably. Secondly, the corresponding algebraic systems are better-conditioned because of using polynomials of smaller degrees. In addition, the algorithms allow for the best use of parallel computers which makes domain decomposition (multi-domain methods more and more popular.

The spectral domain decomposition method was first proposed in [2] in 1980, which it is also one of pioneer work of the spectral element method. Ref. [2] considered the Poisson equation with L-shaped domain. The L-shaped domain was decomposed into three non-overlapping sub-domains. In every sub-domain, the unknown functions were solved by expanding them into Chebyshev series and giving some connection conditions of interface. Such kind of spectral domain decomposition method is also referred to as the spectral element method or multi-domain spectral method. In [2] and [3] two kinds of spectral domain decomposition methods are studied. In the first kind, the domain is divided into several un-overlapped sub-domains and solution is connected through neighbouring interface conditions. This kind includes two cases. One is called spectral element method which is based on Galerkin projection methods. The other is the matching method which is based on collocation points and also called pseudo-spectral element method. In the second kind, they divided the domain into several overlapped sub-domains and solve the equations in stagger in every sub-domain. This kind of domain decomposition method is also known as the Schwarz method.

So far, the study of multi-domain spectral methods is very active and some progress has been achieved in algorithms and error estimates [4, 5] and Quarteroni [6]. Now the multi-domain spectral has been successfully applied into many practical fields such as fluid mechanics, oceanography and aerodynamics [7–9]. The study of spectral domain decomposition method and extending applications have great significance. This might eventually lead to overcoming of the biggest obstacle in the application of spectral method into complex domain problem.

In this paper, we consider the Chebyshev-Legendre spectral domain decomposition method for solving the following two-dimensional vorticity equations

$$\begin{cases} \partial_t\xi+J(\xi,\psi)-\nu\nabla^2\xi=f_1, & x\in\Omega,\ \ t\in(0,T], \\ -\nabla^2\psi=\xi+f_2, & x\in\Omega,\ \ t\in(0,T], \\ \xi(x,0)=\xi_0(x), & x\in\Omega\bigcup\partial\Omega, \end{cases} \tag{1.1}$$

where $I=(-1,1)$, $\Omega=I\times I$, and $x=(x_1,x_2)\in\Omega$, and $\xi=\xi(x,t)$ and $\psi=\psi(x,t)$ are the vorticity function and stream function respectively, $f_i=f_i(x,t)$ $(i=1,2)$ and $\xi_0(x)$ are given functions, $\nu>0$ is the kinematic viscosity, and $J(\xi,\psi)=\partial_2\psi\partial_1\xi-\partial_1\psi\partial_2\xi$ with $\partial_i:=\partial_{x_i}$ $(i=1,2)$. Suppose that the boundary is non-slip and $\psi|_{\partial\Omega}=0$. In addition, similar to the treatment