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A Numerical Study of Complex Reconstruction in Inverse Elastic Scattering

Guanghui Hu[1], Jingzhi Li[2,*], Hongyu Liu[3] and Qi Wang[4]

[1] *Beijing Computational Science Research Center, Beijing 100094, P.R. China.*
[2] *Department of Mathematics, Southern University of Science and Technology, Shenzhen 518055, P.R. China.*
[3] *Department of Mathematics, Hong Kong Baptist University, Kowloon Tong, Hong Kong.*
[4] *Department of Computing Sciences, School of Mathematics and Statistics, Xi'an Jiaotong University, Xi'an, 710049, P.R. China.*

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Abstract. The purpose of this paper is to numerically realize the inverse scattering scheme proposed in [19] of reconstructing complex elastic objects by a single far-field measurement. The unknown elastic scatterers might consist of both rigid bodies and traction-free cavities with components of multiscale sizes presented simultaneously. We conduct extensive numerical experiments to show the effectiveness and efficiency of the imaging scheme proposed in [19]. Moreover, we develop a two-stage technique, which can significantly speed up the reconstruction to yield a fast imaging scheme.

AMS subject classifications: 74J20, 74J25, 35Q74, 35R30

Key words: Inverse elastic scattering, complex scatterers, cavities and rigid elastic bodies, single-shot method.

1 Introduction

This work concerns the numerical realization of an imaging scheme proposed in [19] for reconstructing complex elastic scatterers embedded in a homogeneous isotropic background medium occupying $\mathbb{R}^3$. Let λ and μ be two constants such that $\mu>0$ and $3\lambda+2\mu>0$. λ and μ are the Lamé constants that constitute the parameterization of the background elastic material. Throughout, we assume that the density of the background elastic medium is normalized to be 1. Let $D\subset\mathbb{R}^3$ be a bounded domain with a C^2 boundary ∂D and a connected complement $\mathbb{R}^3\backslash\overline{D}$. D denotes the inhomogeneous elastic body

*Corresponding author. *Email addresses:* `hu@csrc.ac.cn` (G. Hu), `li.jz@sustc.edu.cn` (J. Li), `hongyu.liuip@gmail.com` (H. Liu), `qi.wang.xjtumath@gmail.com` (Q. Wang)

that we intend to recover by using elastic wave measurements made away from it. In what follows, D is referred to as a *scatterer*. The detecting elastic field is taken to be the time-harmonic plane wave of the form

$$u^{in}(x)=u^{in}\left(x;d,d^{\perp},\alpha,\beta,\omega\right)=\alpha d e^{\mathrm{i}k_p x\cdot d}+\beta d^{\perp}e^{\mathrm{i}k_s x\cdot d},\quad \alpha,\beta\in\mathbb{C}, \tag{1.1}$$

where $d\in\mathbb{S}^2:=\left\{x\in\mathbb{R}^3:|x|=1\right\}$ is the incident direction ; the vector $d^{\perp}\in\mathbb{S}^2$ satisfying $d^{\perp}\cdot d=0$ denotes the polarization direction; and $k_s:=\omega/\sqrt{\mu}$, $k_p:=\omega/\sqrt{\lambda+2\mu}$ denote the shear and compressional wave numbers, respectively. Let $u^{sc}(x)\in\mathbb{C}^3$, $x\in\mathbb{R}^3\backslash\overline{D}$ denote the perturbed/scattered elastic displacement field caused by the elastic scatterer and $u:=u^{in}+u^{sc}$ denote the total field. The propagation of the elastic field is governed by the following reduced Navier equation (or Lamé system)

$$(\triangle^*+\omega^2)u=0\quad\text{in }\mathbb{R}^3\backslash\overline{D},\quad \triangle^*:=\mu\triangle+(\lambda+\mu)\text{grad div}. \tag{1.2}$$

In order to complete the description of the direct elastic scattering problem, we next prescribe the physically meaningful boundary conditions satisfied by the elastic field on ∂D and at the infinity.

Define the infinitesimal strain tensor by

$$\epsilon(u):=\frac{1}{2}\left(\nabla u+\nabla u^T\right)\in\mathbb{C}^{3\times3}, \tag{1.3}$$

where ∇u and ∇u^T stand for the Jacobian matrix of u and its adjoint, respectively. By Hooke's law the Cauchy stress tensor relates to the strain tensor via the identity

$$\sigma(u)=\lambda(\text{div}\,u)\mathbf{I}+2\mu\epsilon(u)\in\mathbb{C}^{3\times3}, \tag{1.4}$$

where $\mathbf{I}$ denotes the 3×3 identity matrix. The surface traction (or the stress operator) on ∂D is defined as

$$Tu=T_\nu u:=\nu\cdot\sigma(u)=(2\mu\nu\cdot\text{grad}+\lambda\nu\text{div}+\mu\nu\times\text{curl})u, \tag{1.5}$$

where ν denotes the unit normal vector to ∂D pointing into $\mathbb{R}^3\backslash\overline{D}$. We also define $Ru:=u$ in the following. If D is a cavity, then one has the traction-free boundary condition $Tu=0$ on ∂D; and if D is a rigid body, then one has $Ru=0$ on ∂D.

Decomposing the incident wave u^{in} in (1.1), we denote by $u_p^{in}:=de^{\mathrm{i}k_p x\cdot d}$ the (normalized) *plane pressure wave*, and $u_s^{in}:=d^{\perp}e^{\mathrm{i}k_s x\cdot d}$ the (normalized) *plane shear wave*. By Hodge decomposition, the scattered field u^{sc} can be decomposed into

$$u^{sc}:=u_p^{sc}+u_s^{sc},\quad u_p^{sc}:=-\frac{1}{k_p^2}\text{grad div}u^{sc},\quad u_s^{sc}:=\frac{1}{k_s^2}\text{curl curl}u^{sc},$$