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An Anisotropic Convection-Diffusion Model Using Tailored Finite Point Method for Image Denoising and Compression

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Abstract. In this paper we consider an anisotropic convection-diffusion (ACD) filter for image denoising and compression simultaneously. The ACD filter is discretized by a tailored finite point method (TFPM), which can tailor some particular properties of the image in an irregular grid structure. A quadtree structure is implemented for the storage in multi-levels for the compression. We compare the performance of the proposed scheme with several well-known filters. The numerical results show that the proposed method is effective for removing a mixture of white Gaussian and salt-and-pepper noises.

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Key words: Perona-Malik filter, image denoising, convection-diffusion, tailored finite point method, singular perturbation problem, adaptive grid.

1 Introduction

Image denoising has been one of the most challenging issues since it is very difficult to preserve the edges and the desired textures while removing the noises. For years, many mathematical models for image denoising have been presented, such as the total variation (TV) model by Rudin, Osher and Fatemi [16] and the partial differential equation (PDE) based method pioneered by Perona and Malik [15]. In Perona-Malik (PM) type model, the image is selectively smoothed via the control of the diffusion coefficient depending on the gradient of the pixel intensity values. Thus, the edges as well as the

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desired detailed information can be preserved as the noises are reduced through the evolution of the diffusion processing.

Later on, some nonlinear diffusion filters have been intensively studied in [4, 23]. Due to the elegance and the success of PM model, many nonlinear diffusion filters and algorithms [1, 5, 7, 13, 14, 22] have been proposed for edge-preserving smoothing. Shih *et al.* [20] proposed a convection-diffusion (CD) filter on adaptive grids, and the proposed convection term can quickly reduce the required time steps for reaching an admissible image quality. The CD filter works well especially on a mixture of Gaussian and pepper-and-salt noises in comparing with other PDE based filters.

The TFPM was first proposed by Prof. H. Han and then implemented by Han, Huang and Kellogg [8]. The TFPM performs very well in convection dominated convection-diffusion problems, and the essential concept of the TFPM is the selection of appropriate functions which are particular solutions to the differential equation. As a result, one can derive an approximating difference equation of which the coefficients are solved exactly from the selected functions, and the numerical solution is locally tailored to keep some particular properties of the equation. The TFPM can achieve approximation with higher accuracy even in coarse grid. Due to this special feature, many applications including convection-dominated convection-diffusion-reaction problems [19], first order wave equation [12] and steady magnetohydrodynamics duct flow problem [11] have been successfully implemented. More recently, TFPM demonstrates the effectiveness in solving singularly perturbed problems [9, 10]. Here we will present an anisotropic CD (ACD) filter with the advantage of the TFPM for image denoising.

The paper is organized as follows. Section 2 contains a brief introduction to various nonlinear diffusion filters including the proposed ACD filter for image denoising. In Section 3 we present the TFPM for ACD filter. In Section 4 we extend the TFPM on adaptive grids for the image compression. Section 5 presents the numerical results on several test images. Finally, we give some conclusions in Section 6.

2 Nonlinear filters

2.1 PM diffusion filters

First we briefly review the PM type nonlinear diffusion filter. For a given grayscale noisy image with pixel intensity $u_0(x,y): \Omega \mapsto [0,255]$ the regularized PM type filter presented by Catté *et al.* [4] is

$$\begin{cases} u_t = \operatorname{div}(g(|\nabla(G_\sigma * u)|)\nabla u) & \text{in } \Omega\times(0,T), \\ \dfrac{\partial u}{\partial n} = 0 & \text{on } \partial\Omega\times(0,T), \\ u(x,y,0) = u_0(x,y) & (x,y)\in\Omega, \end{cases} \tag{2.1}$$