

# Delaunay Graph Based Inverse Distance Weighting for Fast Dynamic Meshing

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**Abstract.** A novel mesh deformation technique is developed based on the Delaunay graph mapping method and the inverse distance weighting (IDW) interpolation. The algorithm maintains the advantages of the efficiency of Delaunay graph mapping mesh deformation while it also possesses the ability of better controlling the near surface mesh quality. The Delaunay graph is used to divide the mesh domain into a number of sub-domains. On each sub-domain, the inverse distance weighting interpolation is applied, resulting in a similar efficiency as compared to the fast Delaunay graph mapping method. The paper will show how the near-wall mesh quality is controlled and improved by the new method.

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**Key words:** Dynamic mesh, inverse distance weighting, Delaunay graph mapping.

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## 1 Introduction

Aerodynamic shape optimization, flapping wing and dynamic aeroelastic fluid-structure interaction problems require the interface or boundary to deform during the solution process. In order to propagate the displacement of the boundary into the solution domain, accurate and efficient meshing techniques are needed. There are mainly three types of methods to handle such problems, namely re-mesh, overset mesh and dynamic mesh techniques. The re-mesh techniques regenerate the mesh to account for the deformation of the interface, hence the solution needs to be interpolated from the old mesh to the new mesh. For the aforementioned problems, the mesh needs to be frequently updated,

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which makes the re-mesh approach very time-consuming; furthermore the interpolation between two different meshes also causes extra error for the numerical solution. The overset mesh method is now widely used in simulation of the rigid body motion when large body movement occurs. The dynamic mesh methods use certain techniques to move the mesh nodes according to the deformation. It is generally more efficient than the re-mesh method and the solution interpolation is avoided. The two key issues of this method are the efficiency and mesh quality of the resultant mesh. During the shape deformation process, the mesh needs to be renewed frequently, especially in the dynamic fluid-structure interaction simulation the mesh needs to be updated for each time step, and therefore a fast dynamic mesh technique can efficiently decrease the computational time. The accuracy of the CFD simulation largely relies on the mesh quality, particularly the mesh quality near the wall boundary, since in this region a poor mesh can result in large truncation errors. As a result, high quality and high efficiency are the most important aspects of the dynamic mesh techniques.

Dynamic mesh techniques can be generally classified into two main categories, based on either physical analogy or interpolation [1]. The physical analogy approach, such as the spring analogy approach [2], normally requiring the connectivity information of the mesh, uses certain physics processes to propagate the mesh deformation from the boundary to the solution domain. Though this method has been successfully applied to many unsteady and optimization problems, it is a relatively expensive approach due to the necessary iterations for solving the associated spring systems. For large deformation, it may lead to invalid mesh cells. Farhat *et al.* [3] introduced the torsional springs to prevent the mesh from becoming invalid which greatly improves the robustness of the method. By solving a set of partial differential equations, Loehner and Yang [4,5] developed a method based on elastic analogy. Particularly, by solving a bi-harmonic set of equations, Helenbrook [6] found that both mesh quality and orthogonality are superior to the Laplace equation based method. In general, this type of method is computationally expensive and not suitable for the large deformation problem.

An interpolation method, by applying some interpolation schemes, directly obtains the displacement or the new coordinates of each node. The radial basis function (RBF) method, proposed by de Boer [7], is able to maintain the mesh quality near the boundary. However it needs to solve three large matrices depending on the node number on the surfaces. For practical 3D problems, it becomes very inefficient. Rendall and Allen [8] proposed an approximate RBF method with a data reduction algorithm. It effectively improves the efficiency of the RBF method. On the other hand, this method also introduces surface point mismatching errors. In order to solve this problem, they added a surface correction step [9]. Witteveen and Bijl [10] proposed a direct interpolation strategy by using inverse distance weighting (IDW) interpolation, which is faster than the original RBF method. Luke *et al.* [1] proposed a fast explicit interpolation method based on IDW method, which has a similar mesh quality as the RBF method, but with a relatively faster speed. For large practical 3D problems, these methods still involve substantial computational cost. Liu *et al.* [11] developed a dynamic mesh method based on Delaunay graph