Impact of Local Grid Refinements of Spherical Centroidal Voronoi Tessellations for Global Atmospheric Models

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\textbf{Abstract.} In order to study the local refinement issue of the horizontal resolution for a global model with Spherical Centroidal Voronoi Tessellations (SCVTs), the SCVTs are set to 1024\textsuperscript{2} cells and 4096\textsuperscript{2} cells respectively using the density function. The ratio between the grid resolutions in the high and low resolution regions (hereafter RHL) is set to 1:2, 1:3 and 1:4 for 1024\textsuperscript{2} cells and 4096\textsuperscript{2} cells, and the width of the grid transition zone (for simplicity, WTZ) is set to 18\degree and 9\degree to investigate their impacts on the model simulation. The ideal test cases, i.e. the cosine bell and global steady-state nonlinear zonal geostrophic flow, are carried out with the above settings. Simulation results show that the larger the RHL is, the larger the resulting error is. It is obvious that the 1:4 ratio gives rise to much larger errors than the 1:2 or 1:3 ratio; the errors resulting from the WTZ is much smaller than that from the RHL. No significant wave distortion or reflected waves are found when the fluctuation passes through the refinement region, and the error is significantly small in the refinement region. Therefore, when designing a local refinement scheme in the global model with SCVT, the RHL should be less than 1:4, i.e., the error is acceptable when the RHL is 1:2 or 1:3.

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\textbf{Key words:} Spherical Centroidal Voronoi Tessellation, local refinement, numerical experiments, width of the transition zone, ratio between high resolution and low resolution.

\section{Introduction}

Accurate simulations of global weather and climate require much finer model resolutions. In order to improve the performance of global atmospheric numerical models,
many model developers attempt to refine the model resolution to the greatest extent. However, a finer resolution implies a substantial increase in computational costs. A halving of the horizontal mesh spacing implies an increase in computational cost of about a factor of eight: a factor of four is due to doubling the degrees of freedom in both horizontal directions, and a factor of two is attributed to halving the time step. The computational load associated with increases in global horizontal resolution everywhere quickly exhausts available computational resources. For instance, conducting a global atmospheric simulation with a horizontal resolution of several kilometers is impracticable at present and a daunting computational burden for many institutes. However, if the resolution is only locally refined, obviously the computational resources can be saved. To achieve this goal, multi-resolution (local refinement) schemes will be required [1, 2].

The hierarchical nesting scheme is widely used in limited area structured grid numerical atmospheric models, such as the well-known Weather Research Forecast (WRF) model [3]. WRF supports horizontal nesting that allows the resolution to be fined over a region of interest by introducing an additional grid (or grids) into the simulation. Nested grid simulations can be realized using either 1-way nesting or 2-way nesting. The ratio between the grid resolutions in the low- and high-resolution regions (for clarity, RHL) is generally set to 1:3. However, the local refinement scheme generates reflective waves inside the finer grid boundary, which are not inherent waves of the atmosphere and thus influence the forecast results. At present, many model developers are beginning to pay close attention to local refinement scheme of the global icosahedral mesh model [4, 5]. However, questions remain unanswered how to design the local refinement scheme and what is the appropriate value for the RHL and how wide the transition region between the coarse grid and the fine one should be for the purpose to reduce the simulation errors to minimum.

In this study, we aim to address the above questions. The paper is organized as follows. Section 2 describes the local refinement scheme of the global SCVT model. The ideal numerical experiments and results are presented in Section 3. RHL and the width of the grid transition zone (hereafter WTZ), and shape-preserving issues are discussed in Section 4. Conclusions are summarized in Section 5.

2 Local refinement scheme of the global SCVT model

The local refinement scheme should be designed according to the weather process considered, for instance, the larger the surface wind speed of typhoon is, the finer the resolution should be, and vice versa. When a frontal system is simulated, the resolution over the region of large temperature gradient should be refined. In the present study, two ideal test cases, i.e. Cosine bell and global stable state nonlinear zonal geostrophic flow [9] are chosen because they have analytical solutions. Thus the grid density function corresponding
to the two cases is expressed as [6–8]:

$$\rho(x_i) = (1 - \gamma) \left[ \frac{1}{2} \left( \tanh \left( \frac{\beta}{\alpha} \|x_c - x_i\| \right) + 1 \right) \right] + \gamma.$$  

Here $\beta$ indicates the width of the High Resolution Region (hereafter HRR), $\alpha$ defines the WTZ, $x_c = (0^\circ N, 0^\circ E)$ denotes the center of the HRR and $\gamma$ measures the RHL. For the local refinement grid used in the present study, $\beta$ is set to 30 degrees, which corresponds to the rounded region radius. To discuss the impact of the WTZ, RHL and the different resolutions on the numerical simulation result, the grids with 10242 cells and 40962 cells are selected, and the values of $\alpha$ and $\gamma$ are given in Table 1. To avoid the possible impact of the time step on the simulated results, the time step is set to 100 seconds for all cases.

<table>
<thead>
<tr>
<th>Grid cells</th>
<th>Scheme</th>
<th>$\alpha$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10242</td>
<td>A</td>
<td>$9^\circ$</td>
<td>1:1</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>$9^\circ$</td>
<td>1:2</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>$9^\circ$</td>
<td>1:3</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>$18^\circ$</td>
<td>1:2</td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>$18^\circ$</td>
<td>1:3</td>
</tr>
<tr>
<td>40962</td>
<td>A</td>
<td>$9^\circ$</td>
<td>1:1</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>$9^\circ$</td>
<td>1:2</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>$9^\circ$</td>
<td>1:3</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>$18^\circ$</td>
<td>1:2</td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>$18^\circ$</td>
<td>1:3</td>
</tr>
<tr>
<td></td>
<td>F</td>
<td>$9^\circ$</td>
<td>1:4</td>
</tr>
</tbody>
</table>

The density functions for Scheme A to Scheme F are given in Fig. 1 for 40962 cells. Here Scheme A indicates a uniform grid structure, and the RHL is 1:2 for Scheme B and D and 1:3 for Scheme C and E. WTZs are also different for the above schemes, with $9^\circ$ for Scheme B and C and $18^\circ$ for Scheme D and E. To discuss the impact of the larger resolution ratio on the simulated results, Scheme F is added. Its resolution ratio is 1:4 and WTZ is $9^\circ$. Fig. 1 shows clearly that the six schemes are obviously different. Fig. 2 and Fig. 3 present the resulting meshes of the 11 schemes listed in Table 1.

### 3 Numerical experiments

To discuss the impact of the above-mentioned schemes on the simulated results, two test cases are conducted using the MPAS model [1].
Figure 1: Density function for the Schemes A-F of 40962 cells. The horizontal coordinate indicates the distance (in degrees) from the refinement region center, and the vertical coordinate indicates the value of the grid density function.

Figure 2: Resulting plots for Scheme A-E of 10242 cells.
3.1 Case 1: Cosine bell

We use the case of the cosine bell [9] to verify that it can smoothly pass through the HRR with all of the above-mentioned schemes and we record the resulting errors. The height
and wind fields of the Cosine bell are defined as:

\[
\begin{align*}
h &= \begin{cases} 
\frac{h_0}{2}[1 + \cos(\frac{\pi}{R})], & r < R, \\
0, & r \geq R,
\end{cases} \\
u &= u_0(\cos \varphi \cos \alpha + \sin \varphi \cos \lambda \sin \alpha), \\
v &= -u_0 \sin \lambda \sin \alpha.
\end{align*}
\]

Here \(h_0 = 1000 \text{ m}\), \(r\) indicates the spherical distance between the location \((\lambda, \varphi)\) and the center location \((\lambda_c, \varphi_c)\), with \((\lambda_c, \varphi_c) = (-\pi/2, 0)\). \(r = a \arcsin[\sin \varphi_c \sin \varphi + \cos \varphi_c \cos \varphi \cos(\lambda - \lambda_c)]\) and the radius of the bell is \(R = a/3\), \(\alpha\) indicates the earth radius. The equator speed is \(u_0 = 2\pi a / (12 \text{ days})\), which is approximately 40 m/s, and the parameter \(\alpha\) shows the angle between the solid rotating axis and polar coordinate axis. Here \(\alpha = 0\). It can be found from the cosine bell definition that the simulated bell would return to the departure location after travelling around the earth for 12 days, and the shape will remain unchanged.

### 3.2 Case 2: Global steady state nonlinear zonal geostrophic flow

This test case is about the steady state solution of the nonlinear shallow water equations. Since the initial condition is exactly the solution of the shallow water equations, any biases emerged during the simulation results from discrete representation of the equations. Thereby this case can be used to examine the rationality of each discrete scheme. The initial height and wind fields are written as [9]:

\[
\begin{align*}
gh &= gh_0 - \left(a\Omega u_0 + \frac{u_0^2}{2}\right) \sin^2 \varphi, \\
u &= u_0 \cos \varphi, \\
v &= 0.
\end{align*}
\]

Here the parameters \(\Omega = 7.292 \times 10^{-5} \text{ s}^{-1}\), \(g = 9.80616 \text{ m s}^{-2}\), \(a = 6.37122 \times 10^6 \text{ m}\), \(\varphi\) indicates the latitude, and \(gh_0 = 2.94 \times 10^4 \text{ m}^2 \text{s}^{-2}\), \(u_0 = 2\pi a / (12 \text{ days})\), \(f = 2\Omega \sin \varphi\). The simulation period for this case is 365 days.

### 4 Results and analysis

Numerical experiments are conducted using all of the above-mentioned schemes for the two test cases, and the analysis is focused on the impacts of the RHL and WTZ and the shape-preserving effects of various combinations of the schemes and parameters.

#### 4.1 Impact of the RHL

The Minimal Errors (hereafter MinE) and Maximal Errors (hereafter MaxE) are calculated between the simulation outputs on the 12th and 365th day and the values at the initial time in Cases 1 and 2. Results are listed in Table 2 for 10242 cells and 40962 cells, respectively.
Table 2: Absolute error of each Scheme in Cases 1 and 2.

<table>
<thead>
<tr>
<th>Grid cells</th>
<th>Scheme</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MinE</td>
<td>MaxE</td>
</tr>
<tr>
<td>A</td>
<td>-444</td>
<td>572</td>
<td>-2</td>
</tr>
<tr>
<td>B</td>
<td>-455</td>
<td>590</td>
<td>-4</td>
</tr>
<tr>
<td>10242</td>
<td>C</td>
<td>-505</td>
<td>614</td>
</tr>
<tr>
<td>D</td>
<td>-465</td>
<td>583</td>
<td>-4</td>
</tr>
<tr>
<td>E</td>
<td>-506</td>
<td>622</td>
<td>-10</td>
</tr>
<tr>
<td>A</td>
<td>-187</td>
<td>145</td>
<td>-1</td>
</tr>
<tr>
<td>B</td>
<td>-196</td>
<td>163</td>
<td>-3</td>
</tr>
<tr>
<td>C</td>
<td>-230</td>
<td>230</td>
<td>-4</td>
</tr>
<tr>
<td>40962</td>
<td>D</td>
<td>-199</td>
<td>165</td>
</tr>
<tr>
<td>E</td>
<td>-240</td>
<td>238</td>
<td>-4</td>
</tr>
<tr>
<td>F</td>
<td>-284</td>
<td>332</td>
<td>-5</td>
</tr>
</tbody>
</table>

It can be seen from Table 2 that in both Case 1 and Case 2 with 10242 cells, the absolute values of MaxE and MinE increase when the RHL increases, i.e., the range of errors becomes large. A similar result is also found in Case 1 with 40962 cells. Although the absolute values of MinE and MaxE also increase when the RHL increases in Case 2 with 40962 cells, the increases are not remarkable and remain almost unchanged in certain conditions.

For the grid structure with 10242 cells in Case 1, since the cosine bell theoretically should be back to the initial location on the 12th day, differences in the geopotential height between the 12th day and the first day are calculated for each scheme. Results are presented in Fig. 4, which shows that for Schemes C and E, the RHL of 1:3 produces the largest errors, while the errors produced by Schemes B and D with the RHL of 1:2 are comparable to those in Scheme A with uniform resolution.

The results from the grid structure with 40962 cells, as shown in Fig. 5, are comparable to those from the grid structure with 10242 cells in Fig. 4. However, it is obvious that the error of Scheme F is larger than those of the other five schemes, in other words, when the RHL is down to 1:4, the simulated results are much worse. This result suggests that the larger the RHL is, the worse the simulation becomes.

Fig. 6 displays the result obtained from the grid structure with 10242 cells in Case 2. It can be seen from Fig. 6 that the errors of Scheme C and E are larger with the RHL of 1:3, while the errors of Schemes B and D with the RHL of 1:2 are close to those of Scheme A that has the uniform resolution.

The result obtained from the grid structure with 40962 cells in Case 2 is presented in Fig. 7, which shows that the errors of Scheme F with the RHL of 1:4 are the largest. Schemes B and D with the RHL of 1:2 give errors closest to those of Scheme A that has the uniform resolution, followed by other schemes.
Figure 4: Differences between the potential height on the 12\textsuperscript{th} day and that at initial time for each Scheme (Scheme A, B, C, D and E) in Case 1 with 1024\textsuperscript{2} cells.

The center of the HRR is located at \((0^\circ E, 0^\circ N)\), thereby the front of the cosine bell will begin to enter the HRR on the second day, and the center of the cosine bell will coincide with the center of the HRR on the third day and the rear of the cosine cell will stay in the HRR while the front leaves the HRR on the fourth day. The cosine bell does not overlap with the HRR on other days. Fig. 8 shows the maximal errors of simulated geopotential
height with various schemes compared to the analytical solution for the grid structures with 10242 and 40962 cells respectively in Case 1 as a function of days. It can be seen that for all schemes with 10242 cells, the errors remain unchanged or slightly decrease from the time when the cosine bell enters the HRR to the time when it leaves. However, the errors become larger after the cosine bell leaves the HRR. In addition, note that before the cosine bell enters the HRR, the errors produced by Scheme A that has uniform resolution

Figure 5: Same as Fig. 4, but for 40962 cells.
are smaller than those produced by other schemes with local refinement, while the errors produced by Scheme A are larger than those by other schemes from the 3rd day to the 7th day. The error produced by Scheme A increases much slower than that produced by other schemes from the 8th day to the 12th day. Also the errors produced by schemes with the RHL of 1:2 are smaller than those produced by schemes with the RHL of 1:3.
Figure 7: Same as Fig. 6, but for 4096^2 cells.

For the grid structure with 4096^2 cells, the results are similar to those for the grid structure with 1024^2 cells. For all schemes except Scheme A, the errors remain unchanged or slightly decrease from the time when the cosine bell enters the HRR to the time when it leaves. However, the errors become larger after the cosine bell leaves the HRR. Before the cosine bell enters the HRR, the errors produced by Scheme A with the uniform resolution are smaller than those produced by other schemes with local refinement, whereas the
errors produced by Scheme A are larger than those produced by other schemes from the $4^{th}$ day to the $6^{th}$ day. The error produced by Scheme A increases much slower than those produced by other schemes from the $7^{th}$ day to the $12^{th}$ day. Also, the error produced by Scheme F with the RHL of 1:4 increases the fastest, while the errors from Schemes C and E with the RHL of 1:3 increase faster than those from Schemes B and D with the RHL of 1:2. In summary, the larger the RHL is, the larger the final errors are. However, note that the errors do not increase greatly inside the HRR.

Comprehensive comparison of the results produced by all schemes with the grid structure of 10242 cells to those with the grid structure of 40962 cells indicates that errors of the latter are almost half of the errors of the former. Apparently the higher the resolution is, the smaller the errors are.

### 4.2 Impact of WTZ

From Table 2 it can be seen that the absolute value of MinE increases as the WTZ widens in Case 1 with 10242 cells, but the increase is not significant. When the RHL is 1:2 and the MaxE decreases slightly as the WTZ widens. However, when the RHL is 1:3, the MaxE increases as the WTZ widens. In Case 2 with 10242 cells, when the RHL is 1:2, the absolute value of MinE remains approximately unchanged while the MaxE decreases slightly as the WTZ widens. However, when the RHL is 1:3, the absolute values of MaxE and MinE all increase remarkably. In Case 1 with 40962 cells, the absolute values of MinE and MaxE increase insignificantly with the widening WTZ. In Case 2 with 40962 cells, when the RHL is 1:2, the absolute value of MinE increases slightly and the MaxE remains
almost unchanged as the WTZ grows. While the RHL is 1:3, the absolute value of MinE remains almost unchanged and the MaxE decreases slightly.

Fig. 4 and Fig. 5 indicate that the results from Schemes B and C are respectively comparable to those from Schemes D and E, i.e., the impact of WTZ on the results is negligible. Although Fig. 5 shows that when the WTZ becomes bigger, the errors increase slightly, in general the WTZ has little influence on the results.

Similar conclusions can be obtained from Fig. 6 and Fig. 7, which show that the results from Scheme B are comparable to those from Scheme D except that the error produced by Scheme B is somewhat larger than that by Scheme D. The error resulting from Scheme E is the largest, and the error produced by Scheme C is larger than those by Scheme B and D. The errors from schemes whose RHL is 1:3 and whose WTZ is wider, are bigger, while the errors from schemes whose RHL is 1:2 and whose WTZ is wider are similar to that from Scheme A with the uniform resolution.

Fig. 8 further confirms the results mentioned above that the errors from Schemes B and C respectively are almost the same as that from Schemes D and E before the cosine bell enters the HRR. The impact of the WTZ on the results is small. The results for 40962 cell-grids are similar to those for 10242 cell-grids.

4.3 Shape-preserving

In this section we discuss whether the shape of the cosine bell can be changed or whether there are reflective waves when the cosine bell passes through the HRR. The absolute errors (defined as the difference between the analytical solution and the simulated result) of Scheme A are compared to those of Scheme F for 40962 cells from the 2nd day to the 4th day in Fig. 9. As described in the previous section, the cosine bell enters the HRR on the 2nd day while the center of the cosine bell coincides with that of the HRR on the 3rd day, and the cosine bell is leaving the HRR during the 4th day. In Fig. 9, the left column shows the errors from Scheme F with the RHL of 1:4, and the right column is the same as the left except that it displays the errors from Scheme A with the uniform resolution. The absolute errors in the left column are comparable to those in the right column on the 2nd, 3rd and 4th day. Thereby it is clear that there exists no obvious deformations or reflective waves. In other words, when the cosine bell passes through the HRR, the resulted deformation is negligible. Fig. 8(b) confirms this conclusion.

5 Conclusions

Local refinement of the horizontal resolution has already been applied in limited-area models, such as the one-way nesting or two-way nesting in WRF. However, when the horizontal resolution of a global SCVT is refined to the scale of several kilometers, current computer resources in many institutes cannot meet the demands. Thereby local refinement of horizontal resolution is necessary to be explored for the global SCVT. In this study we examine horizontal resolutions of 10242 cells and 40962 cells, coupled with the
RHLs of 1:2, 1:3 and 1:4 and two WTZs, respectively. The cosine bell and global steady state nonlinear zonal geostrophic flow test cases are simulated and results are analyzed. The primary conclusions are as follows.

1. The larger the RHL is, the larger the resulted errors are. The errors produced when the RHL is 1:4 are significantly larger than those when the RHL is 1:2 or 1:3. Thereby using a RHL of 1:4 is not recommended.
2. Compared to the RHL, the WTZ results in little additional errors. Thus the impact of WTZ is negligible.

3. When the wave passes through the HRR, no significant wave distortion or reflected waves are found, whereas they do exist in the limited-area nested mesh model.

4. The errors produced in the HRR seldom increase. Therefore, when the local refinement algorithm of the global SCVT model is built, the RHL should not be as low as 1:4.

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