Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2017-0110

Vol. **23**, No. 5, pp. 1289-1354
May 2018

Numerical Modelling of Surface Water Wave Interaction with a Moving Wall

Gayaz Khakimzyanov[1] and Denys Dutykh[2,*]

[1] *Institute of Computational Technologies, Siberian Branch of the Russian Academy of Sciences, Novosibirsk 630090, Russia.*
[2] *LAMA, UMR 5127 CNRS, Université Savoie Mont Blanc, Campus Scientifique, F-73376 Le Bourget-du-Lac Cedex, France.*

Received 15 May 2017; Accepted (in revised version) 26 June 2017

Abstract. In the present manuscript we consider the practical problem of wave interaction with a vertical wall. However, the novelty here consists in the fact that the wall can move horizontally due to a system of springs. The water wave evolution is described with the free surface potential flow model. Then, a semi-analytical numerical method is presented. It is based on a mapping technique and a finite difference scheme in the transformed domain. The idea is to pose the equations on a fixed domain. This method is thoroughly tested and validated in our study. By choosing specific values of spring parameters, this system can be used to damp (or in other words to extract the energy of) incident water waves.

AMS subject classifications: 76B15, 76B07, 76M20

PACS: 47.35.Bb, 47.35.Fg

Key words: Wave/body interaction, full Euler equations, movable wall, wave run-up, wave damping, wave energy extraction.

1 Introduction

The mathematical and numerical high fidelity modeling of water waves is a central topic in coastal and naval engineering. The incident waves come and interact with various coastal features. Nowadays the interaction of water waves with bathymetric features is relatively well understood (at least with some special features [49]). A more challenging situation is the interaction of waves with various coastal structures [53]. At this level the most studied situation is the wave/wall interaction and the wall is assumed to be fixed and impermeable. Violent wave impacts have to be modelled in general using more

*Corresponding author. *Email addresses:* `Khak@ict.nsc.ru` (G. Khakimzyanov), `Denys.Dutykh@univ-savoie.fr` (D. Dutykh)

CFD-like methods in the Eulerian mesh-based [9, 20] or Lagrangian particle-based [21] approaches. In the present study we apply the free surface approximation by neglecting all the processes happening in the air above the free surface [10]. In this line of thinking, the interaction of periodic waves with a fixed vertical wall was recently studied in [5] in the framework of a fully nonlinear weakly dispersive wave model [24]. The conditions under which an extreme wave run-up on a vertical wall may happen were describe in [5] as well.

In this study, we focus on wave interactions with a movable vertical wall. The wall motion can be prescribed. In this case we model the wave generation process with a piston-type wave maker [38]. For instance, this problem was considered in the framework of Boussinesq-type equations in [63]. Otherwise, the wall can move under the action of incident waves. We can even assume that a system of horizontal tension/extension springs (with tunable rigidities) is attached to the wall. Thus, the wall may present a certain resistance to the action of waves. This problem can be regarded also as a piston's free motion under the forcing of water waves. The piston mechanical energy conversion and recuperation is a different technological problem which is out of scope of the present study. The extraction of ocean wave energy on industrial scales is not yet a very common practice [33]. However, very active researches in this field are conducted since at least forty years [31, 32]. Consequently, the numerical methods developed below can be used to design and to optimize such Wave Energy Conversion (WEC) devices. Movable walls have been used, for example, in a triplate system proposed back in 1977 by Dr. Francis Farley. The rigidity of strings is chosen to minimize the reflected wave amplitude so that most of the wave energy be converted into the mechanical energy of the device. Mathematical and numerical modeling of this type of WEC devices is considered below. The main point is that (ordinary and partial) differential equations posed in time-varying domains are known to pose notorious theoretical and numerical difficulties [47].

In the present work we consider a two-dimensional non-stationary problem of surface wave motion in a domain with one moving wall. We assume that the wall remains vertical during its motion. Moreover, at least in a vicinity of the moving wall the bathymetry has to be flat to allow its free motion under the action of waves or to follow a prescribed trajectory. The fluid flow is assumed to be potential and we address the complete (i.e., fully nonlinear and fully dispersive) water wave problem [73]. We propose first a reformulation of this problem on a fixed and domain (a square) by introducing a new curvilinear coordinate system. Then, we propose a robust finite difference discretization of this problem with mathematically proven good qualities. The performance of this algorithm is illustrated on several examples of practical interest. In particular, we study the influence of springs rigidity on the oscillatory motion of the wave/wall system.

The results presented in this study can be viewed under a different angle and, thus, they can be applied to another problem of coastal structures protection from wave loads and impacts. In particular, the catastrophic consequences of 2011 Tohoku earthquake and tsunami are widely known nowadays [61]. The protecting structures are made very solid by implementing various security coefficients. However, more economic protections can