Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2017-0032

Vol. **23**, No. 5, pp. 1355-1392
May 2018

High-Order Hybrid WCNS-CPR Schemes on Hybrid Meshes with Curved Edges for Conservation Laws I: Spatial Accuracy and Geometric Conservation Laws

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Communicated by Kun Xu

Received 12 February 2017; Accepted (in revised version) 23 June 2017

Abstract. A type of hybrid WCNS and CPR method for solving conservation laws on hybrid structured and unstructured meshes is proposed. WCNS on structured grid is conjoined with CPR on unstructured grid through CPR on structured grid. The main hybrid technique becomes coupling WCNS and CPR on curvilinear structured grid. Calculation of grid metrics, interpolation methods of physical coordinates and state variables, and computation of Riemann flux near coupling interface are designed to maintain the expected high order accuracy and to satisfy discrete geometric conservation laws in the whole computational domain. Third-order schemes and fifth-order schemes are considered. Numerical simulations show that the proposed hybrid WCNS-CPR schemes can obtain designed accuracy, satisfy geometric conservation law and have good balance of computational efficiency and grid flexibility.

AMS subject classifications: 65D05, 65M06, 65M60, 65M12

Key words: Weighted compact nonlinear scheme (WCNS), Correction procedure via reconstruction (CPR), hybrid schemes, hybrid grid, high-order accuracy.

1 Introduction

High-order methods have received a lot of attention in the CFD community in recent years. According to meshes used, high order schemes can be classified as methods for

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structured meshes, such as WENO-FD [1], WCNS [2], and methods for unstructured meshes, such as WENO-FV [3], DG [4, 5], SD/SV [6, 7], FR/CPR [8–10]. Methods for structured meshes which can use dimensional splitting approach generally have less computational costs and are more efficient than methods for unstructured meshes, but they have difficulty in generating high quality computational meshes for complex geometry. Methods for unstructured meshes can handle complex geometries flexibly but have drawbacks in computational efficiency and have difficulties in efficient multidimensional reconstruction or reliable nonlinear limiters.

To combine the advantages of the two kinds of methods, hybrid methods based on computational domain decomposition have received research interest. In [11], Nakahashi developed a hybrid method between a finite difference method (FDM) and a finite-element method(FEM), which combines the computational efficiency of FDM with the geometric flexibility of the FEM and can be applied to multiple-body flow problems. Costa and Don presented hybrid Spectral-WENO method for nonlinear systems of hyperbolic conservation laws to conjugate the non-oscillatory properties of the high order WENO-FD schemes with the high computational efficiency and accuracy of spectral methods [12]. Léger and Peyret proposed Coupled Discontinuous Galerkin/Finite Difference Solver with discontinuous Galerkin method running on an unstructured mesh and a finite difference method running on a Cartesian grid [13]. Recently, Cheng and Lu et al. introduced multi-domain hybrid DG and WENO methods for conservation laws [14, 15]. The resultant hybrid solver has demonstrated more flexibility in handling complex geometries than a pure WENO-FD scheme and its capacity of saving computational cost compared with a traditional RKDG method. Although those high order hybrid methods have gained many successful results in combining grid flexibility and computational efficiency, they are always based on Cartesian grids and curvilinear grids, which have superiority in generating body-fitted grids, have not been discussed yet. It is not easy for methods on curvilinear grids to satisfy geometric conservation law (GCL). In recent years, many research work about GCL have been done for finite difference schemes [16–18]. In this paper, hybrid schemes on curvilinear grids which can satisfy GCL are developed by combining WCNS on structured grids and CPR on unstructured grids.

WCNS was proposed by Deng et al. [2], which are based on differential equations and point-values. The scheme has merits on compact stencil, simple operating, low computational cost, and can handle shock capturing very well by using nonlinear weights. In addition, with the help of conservative metric method (CMM) [16] and symmetrical conservative metric method (SCMM) [17] to ensure the GCL for high-order finite difference schemes, WCNS becomes applicable in computing flow field around complex geometry [19–22]. But the flexibility is limited because of the difficulty in generating high quality structured grids for complex geometry.

Correction procedure via reconstruction (CPR) method was firstly proposed by Huynh as flux reconstruction (FR) approach [9, 10] for structured grids and then was generalized to unstructured grids by Wang et al. as Lifting Collocation Penalty (LCP) Formulation [8].