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A Direct Imaging Method for Inverse Electromagnetic Scattering Problem in Rectangular Waveguide

Junqing Chen[1,*] and Guanghui Huang[2]

[1] *Department of Mathematical Sciences, Tsinghua University, Beijing 10084, P.R. China.*
[2] *The Rice Inversion Project, Department of Computational and Applied Mathematics, Rice University, Houston, TX 77005-1892, USA.*

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Abstract. We propose a direct imaging method based on reverse time migration (RTM) algorithm for imaging extended targets using electromagnetic waves at a fixed frequency in the rectangular waveguide. The imaging functional is defined as the imaginary part of the cross-correlation of the Green function for Helmholtz equation and the back-propagated electromagnetic field. The resolution of our RTM method for penetrable extended targets is studied by virtue of Helmholtz-Kirchhoff identity in the rectangular domain, which implies that the imaging functional always peaks in the target. Numerical examples are provided to demonstrate the powerful imaging quality and confirm our theoretical results.

AMS subject classifications: 65N21, 78A46

Key words: Electromagnetic waveguide, inverse scattering problem, reverse time migration, extended obstacle.

1 Introduction

Imaging the dielectric obstacles or biological objects in the rectangular waveguides is a quite important subject in the microwave medical imaging and specimen nondestructive testing. In this paper we propose a reverse time migration (RTM) method to find the support of an unknown obstacle embedded in a rectangular electromagnetic waveguide from the measurement of the wave field which is far away from the obstacle (see Fig. 1). Let $\mathbb{R}^3_{ab}=\{(x_1,x_2,x_3)\in\mathbb{R}^3: x_1\in(0,a), x_2\in(0,b), x_3\in\mathbb{R}\}$ be the rectangular waveguide with the rectangular cross section. Denote by $\Gamma_{0a}=\{(x_1,x_2,x_3)\in\mathbb{R}^3: x_1=0 \text{ or } x_1=a, x_2\in(0,b), x_3\in\mathbb{R}\}$ and $\Gamma_{0b}=\{(x_1,x_2,x_3)\in\mathbb{R}^3: x_2=0 \text{ or } x_2=b, x_1\in(0,a), x_3\in\mathbb{R}\}$ the boundaries of $\mathbb{R}^3_{ab}$. Let

*Corresponding author. *Email addresses:* `jqchen@math.tsinghua.edu.cn` (J. Chen), `ghhuang@math.msu.edu` (G. Huang)

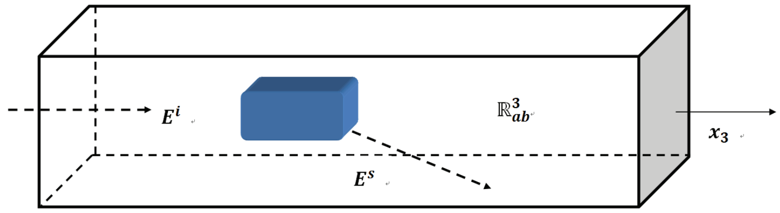

Figure 1: The geometric setting of the electromagnetic scattering problem in rectangular waveguide.

the obstacle occupy a bounded Lipschitz domain D included in $B_\rho=(0,a)\times(0,b)\times(-\rho,\rho)$, $\rho>0$, with ν the unit outer normal to its boundary Γ_D. We assume the incident wave is a point source excited at x_s. The measured wave field satisfies the following equations:

$$\begin{aligned} &\mathrm{curlcurl}E-k^2n(x)E=\delta_{x_s}(x)p && \text{in } \mathbb{R}^3_{ab}, && (1.1)\\ &\nu\times E=0 && \text{on } \Gamma_{0a}\cup\Gamma_{0b}, && (1.2) \end{aligned}$$

and a radiation condition at infinity which will be stated in the following. Where $k>0$ is the wave number, $n\in L^\infty(\mathbb{R}^3_{ab})$ is a positive scalar function and $n(x)-1$ is compactly supported in D, δ_{x_s} is the Dirac source located at $x_s\in\mathbb{R}^3_{ab}$, $p\in\mathbb{R}^3$, $|p|=1$, is the polarization direction of the source. We remark that the results in this paper also apply to non-penetrable obstacles with other boundary conditions on the scatterer.

Now we introduce the radiation condition for completing the systems of (1.1)-(1.2). Let $(\lambda_j^{(1)},\Phi_j^{(1)})$ and $(\lambda_j^{(2)},\Phi_j^{(2)})$ be the complete systems of eigenvalues and orthonormal eigenfunctions in the sense of L_2 of the two-dimensional Laplace operator $-\Delta$ with the Dirichlet and Neumann boundary conditions respectively. For large enough $R>0$ and $|x_3|>R$, we impose the similar condition introduced in [25],

$$\begin{aligned} E(x)=&\sum_j E_j^{1\pm}e^{\mathbf{i}\gamma_j^{(1)}|x_3|}(\lambda_j^{(1)}\Phi_j^{(1)}e_3+\mathbf{i}\mathrm{sign}(x_3)\gamma_j^{(1)}\nabla_2\Phi_j^{(1)})\\ &+\mathbf{i}k\sum_j E_j^{2\pm}e^{\mathbf{i}\gamma_j^{(2)}|x_3|}\nabla_2\Phi_j^{(2)}\times e_3, \qquad (1.3) \end{aligned}$$

where $\nabla_2=e_1\frac{\partial}{\partial x_1}+e_2\frac{\partial}{\partial x_2}$, $\gamma_j^{(p)}=\sqrt{k^2-\lambda_j^{(p)}}$, $p=1,2$, and we choose the branch cut of $\sqrt{z}$ such that $\mathrm{Im}(\sqrt{z})\geq 0$ throughout the paper.

In the last two decades, direct sampling methods have drawn considerable attentions in the field concerning inverse scattering problems. Among these direct methods, the linear sampling method [14], factorization method [16,20] and MUSIC method [2,7] are quite popular. There are also some other direct sampling methods successfully applied to inverse scattering problems in the free space such as [18,19] and the references therein. Due to the multiple scattering by boundaries of waveguide, imaging a scatterer in the waveguide is much more challenging than in the free space [11]. We refer to [6,28,30] for