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An Improved Formulation for Hybridizable Discontinuous Galerkin Fluid-Structure Interaction Modeling with Reduced Computational Expense

Jason P. Sheldon[1], Scott T. Miller[2] and Jonathan S. Pitt[1,*]

[1] *Applied Research Laboratory, The Pennsylvania State University, University Park, PA 16802, USA.*
[2] *Sandia National Laboratories, Albuquerque, NM 87185, USA.*

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Abstract. This work presents two computational efficiency improvements for the hybridizable discontinuous Galerkin (HDG) fluid-structure interaction (FSI) model presented by Sheldon et al. A new formulation for the solid is presented that eliminates the global displacement, resulting in the velocity being the only global solid variable. This necessitates a change to the solid-mesh displacement coupling, which is accounted for by coupling the local solid displacement to the global mesh displacement. Additionally, the mesh basis and test functions are restricted to linear polynomials, rather than being equal-order with the fluid and solid. This change increases the computational efficiency dynamically, with greater benefit the higher order the computation, when compared to an equal-order formulation. These two improvements result in a 50% reduction in the number of global degrees of freedom for high-order simulations for both the fluid and solid domains, as well as an approximately 50% reduction in the number of local fluid domain degrees of freedom for high-order simulations. The new, more efficient formulation is compared against that from Sheldon et al. and negligible change of accuracy is found.

AMS subject classifications: 65M60, 74F10

Key words: Hybridizable discontinuous Galerkin, fluid-structure interaction, HDG FSI, monolithic coupling, arbitrary Lagrangian-Eulerian Navier-Stokes, elastodynamics.

1 Introduction

Many natural phenomena, such as blood transport or aeroelastic flutter [1], require tightly coupled fluid-structure interaction (FSI) simulations to accurately represent the complex multi-physics. FSI simulations, particularly those involving large amplitude, low

*Corresponding author. *Email addresses:* jsp203@psu.edu (J. S. Pitt), jasonpsheldon@gmail.com (J. P. Sheldon), stmille@sandia.gov (S. T. Miller)

frequency, solid deformation, are substantially more computationally expensive and difficult to model than traditional fluid-only or solid-only simulations due to the tight coupling on fluid-solid interface and the disparate mathematics used to describe the fluid and solid regions in the models [2]. With the goal of reducing the computational expense associated with FSI simulations, the hybridizable discontinuous Galerkin (HDG) method was first utilized for FSI simulation by Sheldon et al. [3, 4]. The HDG method is a recently developed finite element method that has the benefits of discontinuous Galerkin (DG) methods, such as scalability in parallel, local conservation of variables, stability with complicated geometries, and high-order accuracy based on the degree of the approximating polynomial [5], while reducing the high computational cost associated with DG methods [6]. This reduction is achieved by separation (hybridization) of the solution into local element solutions and global trace solutions on the element interfaces. This minimizes the number of globally coupled degrees of freedom (DOFs), along with the computational cost of solving the global system, while preserving the benefits of DG methods. Only the primary variable of interest, referred to as the hybrid unknown, exists in both the local and global solution spaces, and it is the only unknown solved for globally, i.e., across the entire mesh, in this formulation. The individual local variables can be solved in an inherently parallel fashion, being completely decoupled from one another.

This work does not focus on deriving formulations with the HDG method or on FSI in general. For more on these topics, please see Sheldon et al. [3,4,7]. Additionally, Nguyen et al. present an excellent introduction to the HDG method in [8] for Stokes flow and in [9] for an overview to a wide variety of physics. Extensive literature exists on a multitude of other computational methods for FSI, e.g. [10–12], and the interested reader is referred to the references therein for further information. This work also does not tackle many of the items identified as future work for HDG FSI in [3,4,7], including: higher order post-processing, the non-linear solid strain's suboptimal convergence rate, parallel processing scaling and efficiency studies, investigation of optimal stabilization parameters for HDG multiphysics, and a full three-dimensional HDG FSI study. These remain interesting and important areas of future work, each worthy of their own investigation.

Instead, the focus of this work is a reduction in the number of DOFs for the HDG FSI system and the resultant increase in computational efficiency over the formulations presented in [3, 4]. Two factors contribute to this DOF reduction. First, the global solid displacement is eliminating from the solid formulation. This has repercussions on the solid-mesh FSI coupling that are discussed later. Second, the function spaces for the mesh test and basis functions are restricted to linear polynomials, regardless of the order of the rest of the simulation. The computational effects of these two changes is a 50% reduction in the number of global DOFs for high-order simulations across both the fluid and solid domains, as well as an approximately 50% reduction in the number of local fluid domain DOFs for high-order simulations. The specific DOF reductions versus simulation order are detailed in Appendix A and Table 4.

In the following section a brief background of the mathematics necessary for HDG FSI modeling is presented. Next, the fluid, solid, and mesh formulations from [3, 4] are