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An Admissible Asymptotic-Preserving Numerical Scheme for the Electronic M_1 Model in the Diffusive Limit

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Abstract. This work is devoted to the derivation of an admissible asymptotic-preserving scheme for the electronic M_1 model in the diffusive regime. A numerical scheme is proposed in order to deal with the mixed derivatives which arise in the diffusive limit leading to an anisotropic diffusion. The derived numerical scheme preserves the realisability domain and enjoys asymptotic-preserving properties correctly handling the diffusive limit recovering the relevant limit equation. In addition, the cases of non constants electric field and collisional parameter are naturally taken into account with the present approach. Numerical test cases validate the considered scheme in the non-collisional and diffusive limits.

AMS subject classifications: 65D, 65C, 76X

Key words: Electronic M_1 moment model, approximate Riemann solvers, Godunov type schemes, asymptotic preserving schemes, diffusive limit, plasma physics, anisotropic diffusion.

1 Introduction

In order to initiate nuclear fusion reactions, it was proposed to use laser pulses in order to ignite a deuterium-tritium target. During this process the energy is transported from the critical surface to denser parts through the electron transport. This transport plays a key role in the understanding of plasma physical phenomena such as, parametric [27, 48] and hydrodynamic [20, 55, 62] instabilities, laser-plasma absorption [36, 53], wave damping [18, 41], energy redistribution and hot spot formation [10, 46]. Spitzer and

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Härm were the first to propose a electron transport theory in a fully ionised plasma without magnetic field. They derived the electron plasma transport coefficients by solving the electron kinetic equation by using the expansion of the electron mean free path over the temperature scale length (denoted ε in this paper). The results of Spitzer and Härm have been reproduced in other works [3,9,56] using the early works of Chapman [15,16] and Enskog [25] for neutral gases. However in the case of non-local regimes [51], the Spitzer-Härm theory is no more valid. Indeed the electron transport plasma coefficients were derived in the case where the isotropic part of the electron distribution function remains close to the Maxwellian. For example, in the context of inertial confinement fusion, the plasma particles may have an energy distribution far from the thermodynamic equilibrium so that the fluid description is not adapted. Moreover kinetic effects like the non local transport [10, 46], wave damping or the development of instabilities [20] can be important over time scales shorter than the collisional time so that fluid simulations are insufficient and kinetic codes have to be considered to capture the physical processes. Therefore, a kinetic description seems unavoidable for the study of inertial confinement fusion processes. However such a kinetic description is accurate but also computationally expensive for describing most of real physical applications. Kinetic codes are often limited to time and length much shorter than those studied with fluid simulations. It is therefore an essential issue to describe kinetic effects by using reduced kinetic codes operating on fluid time scales.

Angular moments models can be seen as a compromise between kinetic and fluid models. On the one hand, they have the advantage to be less computationally expensive than kinetic description since less variables are involved in the models and on the other hand they provide results with a higher accuracy than fluid models [26, 59]. Grad [31], initially proposed a moment closure hierarchy which leads to a hyperbolic set of equations for close equilibrium flows. The hierarchy proposed is based on a polynomial series expansion of a distribution function close to the Maxwellian equilibrium. However, the truncation of this expansion leads to a loss of the positivity of the distribution function and to unrealisable moments, ie moments which can not be derived from a positive distribution function. In [1,43,49,50,58], closures based on entropy minimisation principles are investigated. It has been shown that this closure choice enables to recover fundamental properties such as the positivity of the underlying distribution function, the hyperbolicity of the model and an entropy dissipation property [32, 43, 47]. In this work, the moment model is based on an angular moments extraction. The kinetic equation is only integrated with respect to the velocity direction while the velocity modulus is kept as a variable. The closure used is based on an entropy minimisation principle and gives the angular M_1 model. This model is used in numerous applications such as radiative transfer [5,17,22,52,60] or electron transport [21,34,44]. It satisfies fundamental properties and recovers the asymptotic diffusion equation in the long time and small mean free path regimes [23]. In order to perform numerical simulations, the HLL scheme [37] is often used for the M_1 electronic model because it ensures the positivity of the first angular moment and the flux limitation property. However, this scheme does not degenerate