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Application of the GRP Scheme for Cylindrical Compressible Fluid Flows

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Abstract. This paper contributes to apply both the direct Eulerian and Lagrangian generalized Riemann problem (GRP) schemes for the simulation of compressible fluid flows in two-dimensional cylindrical geometry. Particular attention is paid to the treatment of numerical boundary conditions at the symmetric center besides the zero velocity (momentum) enforced by the symmetry. The new treatment precisely describes how the thermodynamical variables are discretized near the center using the conservation property. Moreover, the Lagrangian GRP scheme is verified rigorously to satisfy the properties of symmetry and conservation. Numerical results demonstrate the performance of such treatments and the symmetry preserving property of the scheme with second order accuracy both in space and time.

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Key words: Euler equations, cylindrical geometry, the generalized Riemann problem (GRP) scheme.

1 Introduction

The simulation of compressible fluid flows in cylindrical symmetric geometry has been received great attention due to practical demands [2, 10, 11] and numerical difficulties as shown in [1, 6, 12, 16–18, 24, 35, 36]. The difficulties lie in the prevention of wall-heating phenomenon near the center [31], the alleviation of the large distortion of Lagrangian mesh [33], the symmetry and positivity preserving [16–18, 35], and the conservation of

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momentum and total energy [28] etc, besides the accuracy requirement [30]. There are quite a few works about this issue in literature. For example, in [16] Cheng and Shu propose a cell-centered Lagrangian scheme with preservation of symmetry and conservation properties, using an equal-angle-zoned initial grid. The area-weighted method is widely used in spherical symmetry preservation in two-dimensional cylindrical coordinates with the sacrifice of strict momentum and total energy conservation [14, 28]. In [35, 36], V*á*chal and Wendroff study a staggered grid Lagrangian scheme to maintain spherical symmetry exactly on an qual-angle-zoned grid. For numerical boundary conditions at the symmetric center, people mostly set zero velocity and use a symmetric extension for thermodynamical variables by using the property of symmetry [16, 35, 36], in addition that the conservation laws are adopted to derive the boundary conditions for spherical cases in [24]. High order WENO type methods are also available [30].

This paper applies the generalized Riemann problem (GRP) solver both in the Eulerian and Lagrangian versions to simulate the cylindrical compressible fluid flows [9, 24]. The GRP solver, a second order temporal-spatial coupled Godunov-type solver, was originally derived in [5] and has been applied extensively since then. This paper contributes the following: (1) The method in [24] is extended for the cylindrical case to derive the boundary condition at the symmetrical center both theoretically and numerically. This newly derived boundary conditions are consistent with the conservation laws of mass, momentum and energy. (2) The geometrical source term is discretized using an interface method [8], for which the interface values by the GRP solver are adopted together with the numerical flux approximation. Such a discretization can keep the well-balancing property, as shown in [8] besides the algorithm simplicity. (3) The symmetry property is automatically preserved due to the feature of the GRP scheme, which is verified rigorously by using the almost the same approach as in [16]. The resulting scheme could be useful even in the study of flow transition problems [15, 21, 22].

We organize this paper as follows. In Section 2, the compressible Euler equations are described both in cylindrical coordinate and local coordinate, the direct Eulerian GRP scheme is given over equal angular polar grid, and the data reconstruction is provided, including gradient computing and limiter constraints. In Section 3, the numerical boundary conditions at the center are derived. In Section 4, we construct a Lagrangian GRP scheme with the symmetry and conservation properties for Euler equations in cylindrical coordinates, and give the numerical boundary condition at the center in Lagrangian formulation. We carry out several numerical examples in Section 5 to demonstrate the accuracy, the efficiency and the performance of the two types of GRP schemes, and the effectiveness of the numerical boundary condition at the center.

2 The GRP scheme for 2-D cylindrical Euler flows

As is well known, the compressible Euler equations in two-dimensional cylindrical geometry are written in the form,